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A Model for Gas Coning and Rate-Dependent Gas/Oil Ratio in an Oil-Rim Reservoir

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Abstract

In oil rim fields a thin oil layer lies between an aquifer and a gas cap. Oil may be produced from such fields using horizontal wells. Production will lower the local gas/oil contact near the well in a process called gas coning (or more accurately, cresting). After gas breakthrough, the gas/oil ratio (GOR) from the well may vary strongly with the production rate. The ability to predict this dependency is essential for production optimisation for such fields.

We have developed a mathematical model that can predict gas coning behaviour and the resulting rate dependent GOR with a surprisingly high degree of accuracy over periods of several months or more. We combine a dynamic model that describes the essential reservoir behaviour with a highly simplified description of the interaction between the well and the surrounding reservoir. The full model has three adjustable parameters that allow us to fit the behaviour to individual wells, using historical oil and gas production rates. The model forms the basis of the GORM (Gas/Oil Ratio Model) computer program that since early 2003 is in regular use for production planning and optimisation at the Troll field. We have also tested the model on wells in other fields, with encouraging results.

Introduction

The Troll Field is located in the North Sea 80 km off the west coast of Norway. It covers an area of 700 km^2. It contains a thin oil layer between a large gas cap and an aquifer. The field consists of three provinces, as shown in Figure 1. In the Troll East Province the oil layer is very thin, so this province has no oil producers. Gas production from Troll East started in 1996. In the Troll West Oil and Gas Provinces the oil layer is between 12 and 24 m thick. The oil here is produced using long horizontal wells. Oil production started in 1995[1-3].

The Troll Oil subsea system is one of the world's largest subsea developments, with more than one hundred wells. Water depths vary from 315 to 340 m.

After gas from the gas cap breaks through into a well, the GOR will be strongly rate-dependent, with GOR increasing with the production rate. The maximum gas processing capacity on the platform limits the total allowable gas production from the oil wells. To maximize oil production in this situation it is essential to know how the GOR from individual wells will change with time and in response to changes in production rate from that well. To our knowledge, no models were available that could successfully predict the rate-dependency of the GOR. We therefore started a research and development activity that resulted in the model described in this paper.

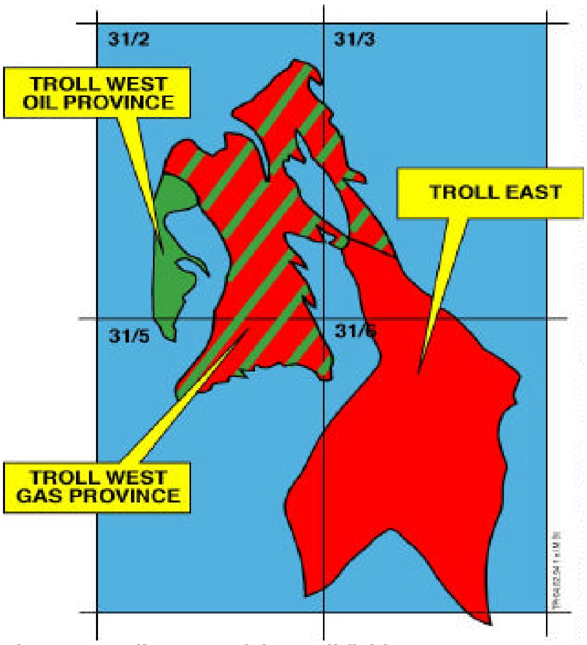

Figure 1: Outline map of the Troll field.

Model description

The Troll reservoir consists of relatively homogeneous sandstone with a very high permeability (1-10 Darcy). The oil viscosity is two orders of magnitude higher than the gas viscosity, so pressure gradients in the gas cap are negligible compared to those in the oil. The oil is produced through sand screens that cover most of the horizontal part of the well. The screen length is typically two to three km, whereas the width of the drainage area for a typical well is 200 to 400 m, i.e. 10 to 20 times the oil layer thickness.

Konieczek[4] exploited these characteristics to construct a simplified model for the oil layer. He used a gravity drainage model where the oil flow towards the well is driven by the hydrostatic pressure gradient in the oil. Only the horizontal flow component normal to the well was modelled. Capillary forces are neglected, thus there is a well-defined gas/oil contact interface (GOC) with no transition zone. The oil-water contact (OWC) is treated as impermeable, a simplification that can be justified as long at the movement of the OWC is limited to localised water coning close to the well. Konieczek's model describes the oil layer thickness h as a function of time t and of the horizontal distance x from the well. The variables are indicated in Figure 2.

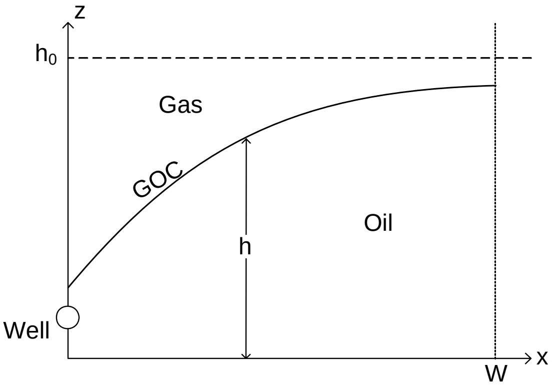

Figure 2: Section normal to the wellbore

We denote the pressure in the gas at level $z = h_0$ by p_0. Assuming hydrostatic equilibrium in the vertical direction, the pressure in the oil column at level z is:

$$p(x,z) = p_0 + \rho_g g(h_0 - h(x)) + \rho_o g(h(x) - z) \qquad (1)$$

Thus, the pressure gradient in the oil column is

$$\frac{\partial p}{\partial x} = (\rho_o - \rho_g) g \frac{\partial h}{\partial x} \qquad (2)$$

The oil flow velocity is given by Darcy's law:

$$u = -\frac{k}{\mu}\frac{\partial p}{\partial x} = -\frac{k}{\mu}\Delta\rho g \frac{\partial h}{\partial x} \qquad (3)$$

Let φ be the effective porosity, i.e. the volume fraction occupied by movable oil. The movable oil volume per unit reservoir area is then:

$$V(x) = h(x)\varphi \qquad (4)$$

The net rate of change of this oil volume is:

$$\frac{\partial V}{\partial t} = \frac{\partial}{\partial x}(uh) = -\frac{kg\Delta\rho}{\mu}\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) \qquad (5)$$

Combining (4) and (5) gives us a partial differential equation (PDE) for h:

$$\frac{\partial h}{\partial t} = \alpha \frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right), \quad \alpha = \frac{k\Delta\rho g}{\mu\varphi} \qquad (6)$$

This equation is known as the Dupuit-Forchheimer equation and is widely used in modelling groundwater flow.

If we denote the volumetric oil production per unit length of well by $\tilde{q}_o$, Konieczek's boundary condition at the well may be written:

$$\tilde{q}_o = -2uh = 2\frac{khg\Delta\rho}{\mu}\frac{\partial h(t,0)}{\partial x} = 2\alpha\varphi h\frac{\partial h(t,0)}{\partial x} \qquad (7)$$

The factor 2 in equation (7) compensates for the fact that the model only covers one half of the reservoir, with the two halves assumed to be symmetrical. We model the outer boundary of the well's drainage area as a no-flow boundary, i.e.

$$\frac{\partial h(t,W)}{\partial x} = 0 \qquad (8)$$

The oil production will lower the gas/oil contact. Ultimately, parts of the wellbore will come in direct contact with the gas cap and start producing gas cap gas along with the oil.

The goal of GORM is to calculate the GOR in this situation. Konieczek did not address this problem. Tiefenthal[5] recreated the GOR from a test well on Troll by combining Konieczek's model for the oil rate with a gas rate that was essentially proportional to the pressure difference between the well and the reservoir. However, the post-breakthrough time interval in Tiefenthal's example was very short. Our attempts at using his approach for modelling longer time series were not successful.

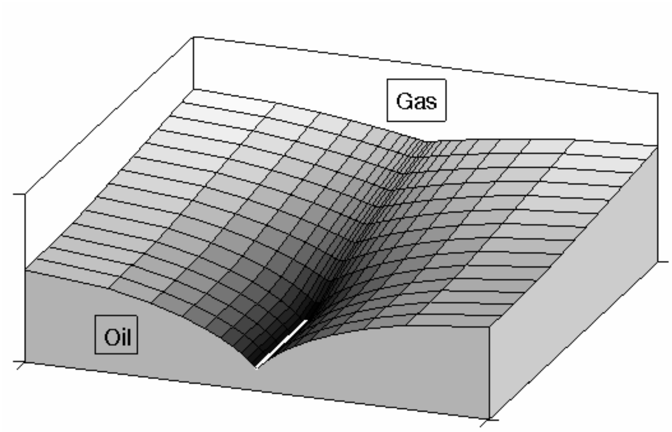

Figure 3: Shape of the gas/oil contact some time after gas breakthrough, as calculated by GORM. The part of the wellbore that is exposed to free gas is shown in white. Not to scale.

Both Konieczek and Tiefenthal based their analysis on a one-dimensional model and thus failed to take variations along the length of the well into account. Usually, the gas breakthrough occurs first at the downstream end of the production interval (the heel). Figure 3 shows how the gas/oil contact may look after gas breakthrough. The length of wellbore exposed to gas will tend to grow but will vary in response to changes in production rate. Large parts of the well will remain covered by oil throughout most of its producing life.

In order to model this behaviour, we introduce the horizontal coordinate, y, in the direction of the wellbore, with y = 0 at the heel at $y = L$ at the toe. We assume that the pressure difference Δp between reservoir and wellbore, commonly known as the *drawdown*, varies linearly with position along the well, from Δp_0 at the heel to $\beta\Delta p_0$ at the toe:

$$\Delta p(y) = \left((\beta-1)\frac{y}{L}+1\right)\Delta p_0 = K(y)\Delta p_0 \qquad (9)$$

Figure 4 illustrates this idealised drawdown profile.

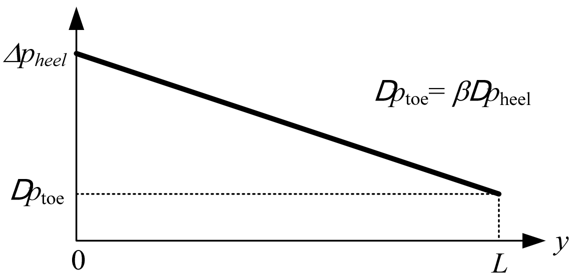

Figure 4: GORM is based on an idealised drawdown profile

In the parts of the well where the oil level $h(t,0,y)$ is below the top of the wellbore we allow free gas from the gas cap to enter the well. The local oil rate is reduced accordingly. The rates per unit well length at point y are given by:

$$\tilde{q}_o = J_o\left(1-\delta^2(y)\right)\Delta p(y) \qquad (10)$$

$$\tilde{q}_g = J_g\delta^2(y)\Delta p(y) \qquad (11)$$

J_o and J_g are the productivity indices per unit well length for oil and gas, respectively. If z_w is the level of the wellbore top and d_w is the wellbore diameter, δ is the dimensionless oil level at the wellbore, defined as:

$$\delta(t,y) \equiv \frac{z_w(y)-h(t,0,y)}{d} \qquad (12)$$

Note that the local ratio of free gas to oil is independent of Δp_0 in this model:

$$\frac{\tilde{q}_g}{\tilde{q}_o} = \gamma\frac{\delta^2}{1-\delta^2}, \quad \gamma = \frac{J_g}{J_o} \qquad (13)$$

The exact form of this relationship appears not to be very important as long as the ratio increases monotonously from 0 at $\delta = 0$ to infinity at $\delta = 1$. The factor γ is a potential tuning parameter.

The relationship between the local gas-oil contact at the wellbore and the local ratio between free gas and oil rates is illustrated in Figure 5.

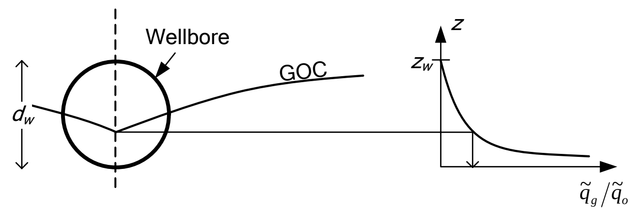

Figure 5: How the local ratio between the production rates of free gas and oil depends on the position of the GOC.

From (10) we get:

$$\frac{\tilde{q}_o}{q_o} = \frac{\tilde{q}_o}{\int_0^L \tilde{q}_o dy} = \frac{\left(1-\delta^2\right)K(y)}{\int_0^L \left(1-\delta^2\right)K(y)dy} \qquad (14)$$

We define ψ by:

$$\psi = \frac{\int_0^L \delta^2 K(y)dy}{\int_0^L \left(1-\delta^2\right)K(y)dy} \qquad (15)$$

The ratio of total free gas to total reservoir oil is given by:

$$\frac{q_g}{q_o} = \frac{\int_0^L \tilde{q}_g dy}{\int_0^L \tilde{q}_o dy} = \frac{J_g}{J_o}\frac{\int_0^L \delta^2 K dy}{\int_0^L \left(1-\delta^2\right)K dy} = \gamma\psi \qquad (16)$$

It might seem natural to use the wellhead choke opening or the downhole pressure as input to the model. However, as indicated by equation (13), the gas/oil ratio can be determined without knowing this pressure. This eliminates the need for modelling the well hydraulics in GORM. Instead we may calculate the gas rate for a given oil rate or vice versa. We have chosen to use gas rate as input and calculate oil rate and GOR. One reason for this choice is model robustness. After gas breakthrough, there is an upper limit to the oil rate the model is able to deliver. If the user specifies a higher rate, the model will fail. On the other hand, the model can handle any gas rate without running into mathematical problems. Gas rate as input is thus the more robust option. This choice also suits the operational practice on Troll, where gas rate is commonly used as the set point for controllers that manipulate the wellhead chokes.

With surface gas rate Q_g as input, the reservoir oil rate q_o is found from equation (17):

$$Q_g \equiv \frac{q_g}{B_g} + \frac{q_o}{B_o}R_s = q_o\left(\frac{\gamma\psi}{B_g}+\frac{R_s}{B_o}\right) \qquad (17)$$

If we know the present position $h(t,x,y)$ of the gas/oil contact we can find ψ. Knowing the observed or planned value of Q_g we can then find q_o from (17) and $\tilde{q}_o$ from (14). Knowledge of the historical and/or planned gas production rate $Q_g(t)$ thus allows us to determine the boundary condition (7), solve the PDE system, and predict the oil rate.

We use a finite volume approach to discretise (6) in the x and y directions and thus transform the PDE to a system of ordinary differential equations (ODE). The shape of the grid is indicated in Figure 3. The ODE system is solved using the ode15s solver[6] in Matlab[7].

Figure 6 shows how GORM can be tuned to follow the observed production history of a typical Troll oil well. The gas production history was used as input to GORM.

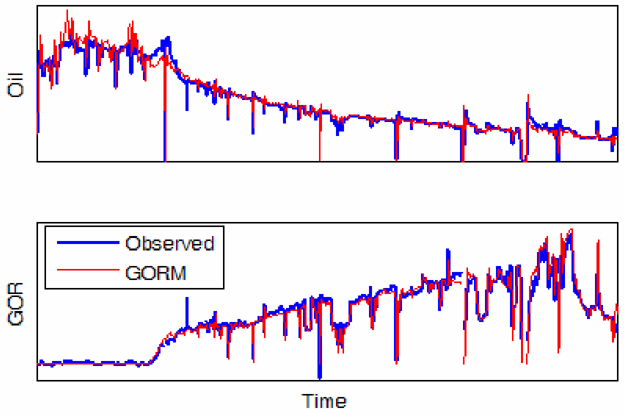

Figure 6: Historic and simulated oil rate and GOR for a typical Troll well over a two-year period from production start. Note how the GOR increases and the oil rate starts to decline after gas breakthrough at approximately five months.

Rate dependent GOR

The main goal of the model development was to be able to predict how the Gas/Oil Ratio varies with production rate for the individual wells. Since the model uses the gas rate as input, we need to find the oil rate as a function of the gas rate. We find this relationship at a given time by simulating the response to a set of alternative step changes in gas rate. Each step starts from the same state, i.e. the same shape of the GOC surface. Figure 7 illustrates how we find the rate-dependency at t = 1000 days from production start, for an ideal simulation using a constant gas rate up to this time.

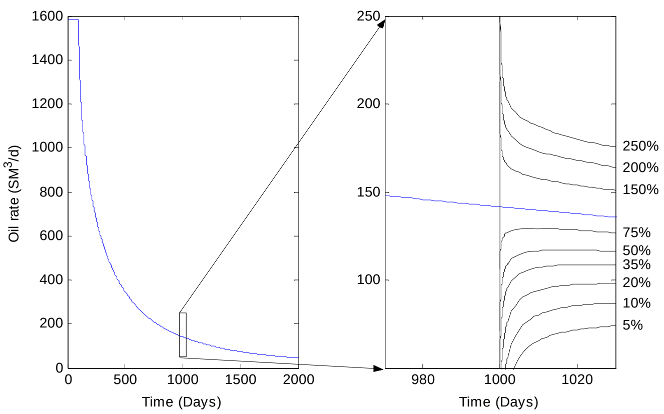

Figure 7: Left: Oil rate vs. time for constant gas rate. Right: Oil rate response to nine alternative step changes in the gas rate at t = 1000 days. The percentages indicate the gas rate after the step, relative to the base rate.

The response curves give us the information needed for planning and optimisation purposes. The oil rates never stabilise completely, so we normally use the mean rates over a fixed interval, e.g. 30 days. Figure 8 shows curves for GOR and marginal GOR ($R_m = dQ_g / dQ_o$) at three different times. The significance of the marginal GOR for maximising oil production is discussed in the chapter on applications, below.

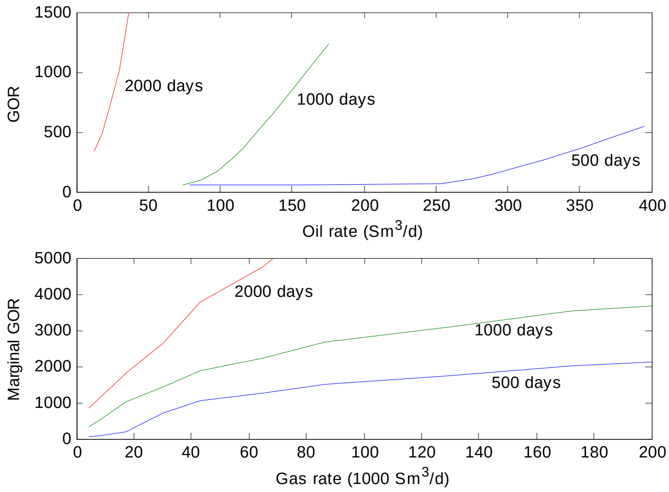

Figure 8: GOR vs. oil rate and marginal GOR vs. gas rate at three different times.

The underlying cause of the rate dependence is that the oil rate from the parts of the well that are exposed to the gas cap gas is limited by the gravity-driven flow of oil towards the well. An increase in drawdown will increase the oil production from those parts only marginally, whereas the gas rate is approximately proportional to drawdown. Thus, variations in drawdown will influence gas rates more than it influences oil rates, leading to rate-dependent GOR. The sensitivity of the oil rate to drawdown will decrease as the length of wellbore covered with oil decreases. The sensitivity of GOR to production rate will increase correspondingly.

GORM captures this behaviour in more detail than reservoir simulators traditionally do. The main reason for this is probably that we follow the vertical movement of the interface between oil and gas more precisely than is commonly done. Furthermore, by modelling one well at a time and by blatantly disregarding known complications such as reservoir inhomogeneities, the history-matching problem becomes quite manageable. This also allows us to align the grid with the wellbore. Many of the simplifications used in GORM may of course not be applicable to more complex reservoirs.

Tuning of the model

We assume that the reservoir within the drainage area of the well has the form of a rectangular parallelepiped ("a shoebox") with the oil-water contact as its base and the original gas-oil contact at the top. The well follows a curve whose projection in the horizontal plane is a straight line running down the centre of the reservoir. The vertical well position may be a function of y. We assume that the two halves of the reservoir are mirror images at all times, so we model only one half.

If we model the production section of the well as perfectly horizontal, only twelve parameters are needed to define a GORM model (see the Nomenclature section for explanations of the symbols):

Well geometry:	L, z_w, d_w
Fluid properties:	B_o, B_g, R_s
Reservoir properties:	α, φ, W, h_0
Miscellaneous:	β, γ

Most of the above parameters are determined from readily available information about the well and the reservoir. The model is tuned to historical production data by adjusting the parameters α, β, and W. Permeability and fluid data may be used to find a good starting value for the tuning of α. The gas/oil productivity ratio, γ, is a possible fourth tuning parameter, but this complicates the tuning process with only a negligible gain in accuracy. Therefore we have chosen to fix γ at a nominal value.

We use the historical gas rate as input and vary the three tuning parameters to get the best possible fit of the predicted oil rate to the historical values, weighting the newest history more than older values. The tuning can be done automatically, but the user must critically evaluate the results.

The numerical solution procedure needs a few more parameters, but they are not relevant for this description of the model.

Modelling results

GORM models have been tuned to nearly one hundred wells on the Troll field. The results vary from excellent to quite poor, with good or acceptable results for a majority of the wells. Figure 9 shows GORM predictions for one well during a ten-month interval after tuning.

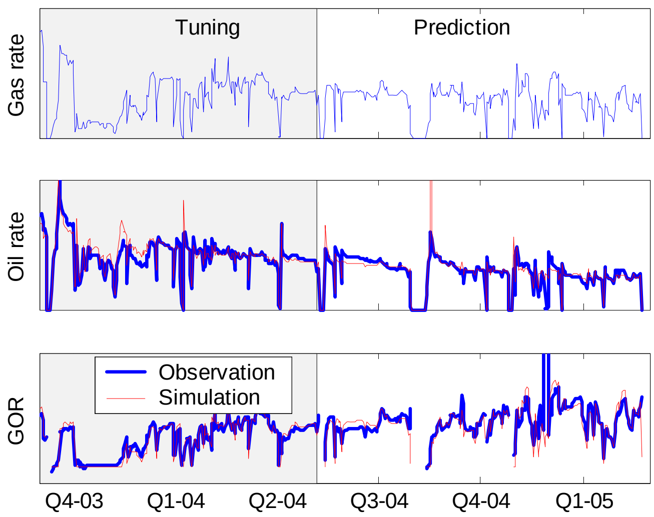

Figure 9: Comparison of simulated (red) and observed behaviour (blue) for one well. The model parameters were determined in May 2004. The simulation shown was done in March 2005, using the full gas rate history as input to GORM. The simulation did not make use of any information about observed oil rates or GOR during the prediction period.

Limitations of the model

The simplifications in the model inevitably limit its usefulness for some purposes. The assumption of purely horizontal oil flow is clearly not valid close to the well. Thus, the GOR from the well may sometimes change much faster than predicted by the model. In particular, the model overestimates the time to new gas breakthrough after a production stop. This has the effect that the predicted oil rates during the first few days after a stop are much higher than observed. In Figure 9 this effect is responsible for a spurious peak in the predicted oil rate after the stop in the third quarter of 2004.

Other factors that may limit the applicability of the model include large-scale movement of the OWC and major flow obstructions in the reservoir, such as shale layers or faults.

Applications

Assume that the total allowable gas production from a group of wells is limited and that we want to allocate production capacity to the individual wells in such a way that we obtain the maximum total oil rate while honouring the constraint on gas. As pointed out by Urbanczyk and Wattenbarger[8], if there are no other active constraints, then at the maximum all wells must have the same marginal GOR. The marginal GOR is defined as the derivative of gas rate with respect to oil rate. If good GORM models exist for all wells, then it is straightforward to find the optimum allocation pattern from the curves of marginal GOR for each well.

On Troll the total gas processing capacity is an active constraint most of the time. I addition several other constraints may be active: The hydraulics of the combination of wells and the production line from a subsea well cluster to the platform may limit the production rate from the cluster. Water processing capacity may act as another constraint. Hauge and Horn[3] describe how GORM is used in combination with the production optimisation tool GAP[9] to plan and optimise production from the Troll Field, taking all relevant constraints into account.

The model has also been used to generate decline curves for long-term planning.

Conclusion

GORM is a mathematical model of the dynamic gas coning behaviour around horizontal oil wells in the Troll field. It combines a dynamic model describing the essential reservoir behaviour and a highly simplified representation of the interaction between the well and the reservoir. The model has been tuned to oil wells on the Troll field with surprisingly good results. Tuned models can predict the future GOR and the sensitivity of the GOR to changes in production rate. The predictions of rate-dependent GOR provided by GORM are essential for the successful optimisation of oil production. In many cases the predictions have proved accurate over periods of several years.

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Nomenclature

B	Formation volume factor
d	Diameter
g	Gravitational acceleration
h	Oil layer thickness
J	Productivity index per unit well length
k	Permeability
K	Drawdown modifier
L	Length of production interval
p	Pressure
q	Volume flow rate (reservoir conditions)
$\tilde{q}$	Volume flow rate per unit well length
Q	Volume flow rate (standard conditions)
R_m	Marginal gas/oil ratio (dQ_g/dQ_o)
R_s	Solution gas/oil ratio
t	time
u	Flow velocity
V	Oil volume per unit reservoir area
W	Drainage area half-width
x	Horizontal coordinate normal to wellbore
y	Horizontal coordinate along wellbore
z	Vertical coordinate
α	Oil level diffusion velocity (eq. 6)
β	Toe/heel drawdown ratio
γ	Gas/oil productivity ratio
δ	Dimensionless oil level (eq. 12)
μ	Viscosity
ρ	Density
$\Delta\rho$	Oil - gas density difference
φ	Effective porosity
ψ	Oil inflow coefficient

Subscripts

0	Reference or initial
g	Gas
o	Oil
w	Wellbore

Units

Any consistent set of units may be used.

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