Chapter 9

Flow separation and flow attachment

We have already postulated that the flow in the boundary layer may separate and set out from the boundary, so that the nonviscous flow becomes restricted to a volume less than what is geometrically maximal. In this chapter we will give an empirical description of the separation phenomenon and its opposite, flow attachment, and also use the equations of motion to deduce conditions for separation to appear.[1]

9.1 A qualitative common trait

Figure 9.1 shows a 2D flow past a symmetrical block on a wall, at small and large $\mathcal{R}$. Despite a considerable qualitative difference of behavior,they have one trait in common: A *flow separation* at a *separation point* S on the wall upstream of the block, with retrograde flow at the opposite side of S, as indicated in Figure 9.2. Boundary layer separation ($\mathcal{R} \gg 1$) may thus be considered as a special case of flow separation, which is a more general phenomenon. Since $u(y = 0) = 0$, the distinction between the flow upstream and downstream of S is in $\operatorname{sgn} \partial u/\partial y\,|_{y=0}$:

- A separation point for flow in the x direction has $\frac{\partial u}{\partial y}\,|_{y=0} = 0$

9.2 Conditions for flow separation

In what follows we will consider a simplified situation, which is adequate for presenting the basic behavior:

- Stationary 2D flow,, $\hat{\boldsymbol{e}}_x$ parallel to and $\hat{\boldsymbol{e}}_y$ normal to a wall
- Arbitrary $\mathcal{R}$

The vorticity will have only one component, which may be treated as a scalar:

$$\boldsymbol{u} = (u, v, 0) \tag{9.1}$$
$$\boldsymbol{\omega} = (0, 0, \zeta) \tag{9.2}$$

[1] It is largely based on the presentation in [Tritton 1988].

Figure 9.1: Streakline patterns for 2D flow at a) $\mathcal{R} = 0.02$ and b) $\mathcal{R} = 3300$

The definition $\zeta = \partial_1 u_2 - \partial_2 u_1 = \partial v/\partial x - \partial u/\partial y$ implies:

$$\frac{\partial \zeta}{\partial y} = \frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} \tag{9.3}$$

At the wall ($y = 0$), because of the boundary conditions $u = v = 0$ for alle x:

$$\frac{\partial u}{\partial x}|_{y=0} = \frac{\partial v}{\partial x}|_{y=0} = 0 \tag{9.4}$$

Using the continuity equation (**??**) we get:

$$\frac{\partial v}{\partial y}|_{y=0} = \frac{\partial^2 v}{\partial x \partial y}|_{y=0} = 0 \tag{9.5}$$

Thus, for the vorticity at the wall the following condition holds:

$$\zeta = -\frac{\partial u}{\partial y} \qquad (y = 0) \tag{9.6}$$

$$\frac{\partial \zeta}{\partial y} = -\frac{\partial^2 u}{\partial y^2} \qquad (y = 0) \tag{9.7}$$

Figure 9.2: Typical flow pattern at point of separation

Figure 9.3: Retrograde circulation and reversed vorticity

This implies that the vorticity at the wall *changes sign* at the separation point. Thus, separation implies the existence of a region where the vorticity has the opposite sign of compared to elsewhere in the flow. For a flow situation like that in Figure 9.2,[2] it means that a*positive* ζ is present at flow separation. However, the region with $\zeta > 0$ has to be less than the total region with retrograde circulation. This is illustrated in Figure 9.3, where the vorticity in the standing eddy's center necessarily has the same sign as upstream of the separation point.

The considerations above show that in the x componenet of the Navier-Stokes equation,

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2} + \nu\frac{\partial^2 u}{\partial y^2}$$

only two terms will be different from zero infinitesimally close to the wall (irrespectively of whether separation occurs or not):

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} = 0 \tag{9.8}$$

[2]The z axis points out of the paper, in the direction of the reader's eyes.

Accordingly:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial \zeta}{\partial y} = 0 \tag{9.9}$$

We have found a relation between the pressure gradient along the wall and the vorticity gradient normal to the wall. It is a *general* relation in the sense that no assumptions about $\mathcal{R}$ have been made; the relation holds for creeping flow as well as boundary layer flow.

With flow and vorticity given by (9.1) and (9.2), we have $(\boldsymbol{\omega}\cdot\nabla)\boldsymbol{u} = 0$. The remaining non-disappearing component of the vorticity equation (**??**) simplifies to

$$(\boldsymbol{u}\cdot\nabla)\zeta = \nu\nabla^2\zeta \tag{9.10}$$

As mentioned in the discussion in Chapter 2, and analogous to properties known from usual diffusion equations, the term $\nu\nabla^2\zeta$ describes diffusion of vorticity down a vorticity gradient, in the direction of smaller absolute values of the vorticity.

If, for the choice of reference system in our example, there may be any regions with a positive ζ out in a flow region with otherwise predominantly negative ζ, it follows that there must be a region for $y = 0$ (where vorticity is created) with $\zeta > 0$ and $\partial\zeta/\partial y < 0$, to get diffusion of positive vorticity out from the wall. It follows from Eq. (9.9) that

- $\partial p/\partial x\,|_{y=0} > 0$ is a necessary, but not sufficient, condition for flow separation (for flow in the positive x direction)

We have thus found an argument for the boundary layer classification which was introduced in the previous chapter. For $\mathcal{R} \gg 1$ we have $\partial p/\partial x \approx \partial p_0/\partial x$ (Λ neglisjerbar), so that an unfavorable pressure gradient implies the possible existence of regions with positive ζ in the flow, and accordingly flow separation.

Whether flow separation actually occurs if the condition above is satisfied, will depend on the full vorticity balance as expressed in Eq. (9.9). Even for $\partial p/\partial x\,|_{y=0} > 0$, diffusion of vorticity from regions with $\zeta < 0$ may counteract the presence of positive ζ in the flow, so that separation will not take place.[3] The eventual presence of separation will depend on two things:

- Whether the flow develops a region at the wall with an unfavorable pressure gradient
- Whether this gradient is unfavorable enough

To determine whether the latter is satisfied is usually difficult. Both aspects depend on the type of flow, and thus on $\mathcal{R}$.

Figure 9.4 shows some idealized velocity profiles elucidating the above relations. The vorticity distribution has some similarities with the distribution of $-\partial u/\partial y$ for small y, with identity in the limit $y \to 0$. The form of the curves is determined by the sign of $\partial^2 u/\partial y^2$ at the wall; if it is positive, it implies that the velocity distribution must have an *inflection point* at some distance from the wall.[4] If $\partial^2 u/\partial y^2\,|_{y=0}$ (and thus also the pressure gradient) is large enough, it means that a region with retrograde flow may occur at the wall. In Figure 9.4d) we see such an explicit signature of separation; the profile may stem from a point downstream of a separation point.

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[3]Without the diffusion mechanism, however, there would not be any mechanism to create regions with a positive ζ.

[4]In the next chapter we will see that an inflection point in the velocity profile implies the existence of an instability of the laminar flow at large $\mathcal{R}$, corresponding to the *Kelvin-Helmholz instability*. However, that instability is not related to the separation phenomenon.

Figure 9.4: Velocity profiles and corresponding velocity gradient profiles for a) favorable pressure gradient, b) zero pressure gradient (the Blasius profile), c) and d) unfavorable pressure gradient

9.3 Separation of creeping flow

Figure 9.5 shows an example. As in Figure 9.1a) the streamline pattern is symmetric, in addition to the symmetric geometry of the boundaries. This is due to the reversiblilty of the flow, as discussed in Chapter 3. The reversibility predicts retrocirculating regions with flow separation both upstream and downstream, as also observe in the figure.

One may visualize that the emergence of *two* regions in Figure 9.5 with retrocirculation as follows: The viscosity brings about that for a flow to pass between the cylinder and the wall, a pressure decrease *larger* than the typical pressure decrease at the cylinder is needed there. At the wall upstream of the passage the pressure has to increase to make that happen; downstream it has to increase once again because the flow near the wall has experienced a decrease larger than the typical decrease, while passing between the cylinder and the wall.

9.4 Boundary layer separation

At large $\mathcal{R}$ the flow is divided into

- nonviscous zones
- boundary layer zones

and flow separation is associated with the latter.[5]

Such separation may occur for instance at the wall in a diverging channel, due to pressure increases described by Euler's equation. If a wall has a sharp edge, separation will usually take place there.

Laminar as well as turbulent boundary layers may separate. (For the latter, the streamlines in Figure 9.2 will denote the averaged flow.) Usually, turbulent boundary layers will need the largest distance with an unfavorable pressure gradient before separation takes place.

[5] The previous analysis in this chapter did not include stagnation points. They are not classified as separation points.

Figure 9.5: Streakline pattern in a 2D flow past a cylinder near a wall, at small $\mathcal{R}$

Figure 9.6: Flow past a wing profile at large angle of attack

The flow pattern for boundary layer separation will depend strongly on the type of flow. Figure 9.6 shows an example at a stall (see Chapter 7). Over the wing, downstream of the separation point, there will be a fluctuating retrocirculating flow.

9.4.1 Boundary layer separation and drag

In a flow with separation as a significant phenomenon, vorticity produced in the boundary layer becomes transported out into regions originally assumed to be irrotational. The presence of separation may modify the nonviscous flow to a considerable degree, in particular concerning the pressure distribution. d'Alembert's principle loses its validity more due to a changed pressure distribution than due to surface friction.

As an example, in Figure 9.7 we see the pressure distribution at a cylinder in flow. In a large $\mathcal{R}$ region, including the value used in the figure, separation will take place a little upstream of the largest cylinder width. The pressure asymmetry due to the vorticity produced, will give

Figure 9.7: Pressure distribution at a cylinder. Broken line: Nonviscous solution; solid line: Measured distribution for $\mathcal{R} = 1.9 \times 10^5$

rise to the drag force.

As known from the introduction to dimensional analysis in basic courses, the drag coefficient C_D, defined as in Eq. (**??**), is roughly independent of $\mathcal{R}$ when the viscosity is not a 'relevant' parameter. The irrelevance may be interpreted as follows: If the point of separation stays roughly fixed, then pressure differences will be proportional to the squared velocity according to Bernoulli's equation, and so C_D will stay roughly independent of $\mathcal{R}$. Jumps in the value of C_D occur when the point of separation moves significantly, for instance when the boundary layer becomes turbulent. See Figure 9.8, where the result of the transition to separation of a turbulent boundary layer is seen as a jump towards *lower* values of C_D at $\mathcal{R} \sim 4 \times 10^5$. The lower value occurs because the region with a changed pressure distribution will cover a smaller fraction of the cylinder area, when the separation point moves downstream. The effect of the type of flow on the drag coefficient is also shown for a sphere[6] in Figure 9.9, where sand grains glued to the sphere's surface make the boundary layer turbulent.

For *streamlined* bodies, separation can be delayed or removed altogether, except at the downstream end where the boundary layers meet and form the wake; see Figure 9.10. Despite a larger surface area and larger viscous drag, the absence of large pressure differences can make C_D become as small as 1/15 of the value for a cylinder with the same thickness.

9.5 Attachment and reattachment. The Coanda effect

As shown already in several figures in this chapter, also *flow attachment* may occur, a phenomenon being the opposite of flow separation. At small $\mathcal{R}$, reattachment may occur as a consequence of the reversibility of the flow. At large $\mathcal{R}$ we talk about the *Coanda effect*, the

[6]In much of the technical literature, C_D is defined by introducing the actual area instead of the square of a characteristic length. As mentioned in another footnote in Chapter 7, that would correspond to multiplying the value of 'our' C_D for a sphere (but not for a cylinder) by $4/\pi$. In Figure 9.9 such modified C_D values have been used.

Figure 9.8: Drag coefficient as a function of $\mathcal{R}$ for a sylinder

Figure 9.9: Separation point on a sphere at $\mathcal{R} \sim 10^6$, for a) smooth surface and b) rough surface on the sphere's upstream end

Figure 9.10: Flow past a) streamlined body and b) cylinder

Figure 9.11: Attachment at large $\mathcal{R}$: a) Coanda effect, b) boundary layer reattachment

tendency of a free jet to entrain the adjacent fluid. If such a jet is close to a wall, pressure effects may make the jet suck itself ttowards the wall; see Figure 9.11a.[7] Turbulent flow has a stronger entrainment than laminar flow (see a later chapter), and a stronger tendency for a Coanda effect to occur. The effect is usually stronger in 2D flow than in 3D flow; in the latter case, inflow from the sides may lessen the unwanted pressure effects between the jet and the wall.

The entrainment process may also occur for separated boundary layers, and may cause reattachment of the boundary layer as indicated in Figur 9.11b, provided the unfavorable pressure gradient does not reach far enough. This may happen without a transition to a turbulent boundary layer. Often, however, a laminar boundary layer flow will become turbulent just after separation, which may make it attach to the wall again. See Figure 9.12, where a subsequent turbulent separation is also shown.

9.6 Problems

Problem 9.1 Consider flow of a fluid with density ρ and nonviscous velocity u_0 along a wall with length L. Close to a separation point it must be possible to have satisfied both (1) the Bernoulli equation and (2) the relation $u^2(y) < \Delta u_0^2$. Show that the minimal order of magnitude of the pressure increase Δp (corresponding to Δu_0^2) over a distance Δx

[7]In 1987/1988, at a local institute for applied research, an 'accelerated test' of an injection valve on a well injection pipe was to be performed. The injected fluid was assumed to mix into the fluid flowing in the annulus, and sand was added to it to increase the friction. However, a Coanda effect occurred: The jet attached to the outer pipe and ground a hole in the wall! Fortunately, this took place in a surface test stall, and not in downhole test facilities.

Figure 9.12: Separation points at a cylinder, for various $\mathcal{R}$ values

along the wall, for a boundary layer separation to occur, is given by

$$\Delta p \sim \rho u_0^2 (\frac{\Delta x}{L})^{2/3}$$