Oppgaveløsninger til kapittel 8

8.1

a) Partikkel, $s=1$, har $l=0$; hvilke mulige j-verdier?

$$\left.\begin{aligned} |l-s| &= |0-1| = 1 \\ l+s &= 0+1 = 1 \end{aligned}\right\} \qquad \underline{j=1}$$

b) Samme partikkel, $l=3$; hvilke mulige j-verdier?

$$\left.\begin{aligned} |l-s| &= |3-1| = 2 \\ l+s &= 3+1 = 4 \end{aligned}\right\} \qquad \underline{j = 2, 3, 4}$$

8.2

a) Partikkel, $s=\frac{3}{2}$, $l=1$; hvilke mulige j-verdier?

$$\left.\begin{aligned} |l-s| &= \left|1-\tfrac{3}{2}\right| = \tfrac{1}{2} \\ l+s &= 1+\tfrac{3}{2} = \tfrac{5}{2} \end{aligned}\right\} \qquad \underline{j = \tfrac{1}{2}, \tfrac{3}{2}, \tfrac{5}{2}}$$

b) Hvilke m_j-verdier for hver j-verdi?

$$j=\tfrac{1}{2}: \quad m_j = \underline{-\tfrac{1}{2}, \tfrac{1}{2}}$$

$$j=\tfrac{3}{2}: \quad m_j = \underline{-\tfrac{3}{2}, -\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{3}{2}}$$

$$j=\tfrac{5}{2}: \quad m_j = \underline{-\tfrac{5}{2}, -\tfrac{3}{2}, -\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{3}{2}, \tfrac{5}{2}}$$

8.3

$S_x = \frac{1}{2}\hbar\begin{pmatrix}0&1\\1&0\end{pmatrix}$ OK $\qquad S_y = \frac{1}{2}\hbar\begin{pmatrix}0&-i\\i&0\end{pmatrix}$ OK $\qquad S_z = \frac{1}{2}\hbar\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ OK

$S_+ = \frac{1}{2}\hbar(\sigma_x + i\sigma_y) = \hbar\begin{pmatrix}0&1\\0&0\end{pmatrix}$ ikke hermitisk

$S_- = \frac{1}{2}\hbar(\sigma_x - i\sigma_y) = \hbar\begin{pmatrix}0&0\\1&0\end{pmatrix}$ " "

Men derimot $S_\pm^\dagger = S_\mp$!

8.4

$$S_y - cI = 0 \Rightarrow \begin{vmatrix}-c & -i\frac{\hbar}{2}\\ i\frac{\hbar}{2} & -c\end{vmatrix} = 0, \quad c^2-(-i)i\left(\frac{\hbar}{2}\right)^2 = 0, \quad c = \pm\frac{\hbar}{2}$$

$$\frac{\hbar}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix} = \pm\frac{\hbar}{2}\begin{pmatrix}\psi_1\\\psi_2\end{pmatrix}$$

Øvre fortegn: $\left.\begin{aligned} -i\psi_2 &= \psi_1 \\ i\psi_1 &= \psi_2 \end{aligned}\right\}$, ett valg er $\begin{pmatrix}1\\i\end{pmatrix}$, $|\nearrow\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$

Nedre fortegn: $\left.\begin{aligned} -i\psi_2 &= -\psi_1 \\ i\psi_1 &= -\psi_2 \end{aligned}\right\}$, ett valg er $\begin{pmatrix}1\\-i\end{pmatrix}$, $|\swarrow\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$

stemmer med (8.24)/(8.25)

8.11

Spinn-$\frac{1}{2}$ partikkel, $|\psi\rangle = \sqrt{\frac{2}{3}}|\uparrow\rangle + i\sqrt{\frac{1}{3}}|\downarrow\rangle = \begin{pmatrix} \sqrt{\frac{2}{3}} \\ i\sqrt{\frac{1}{3}} \end{pmatrix}$

a) Normalisering:

$$\langle\psi|\psi\rangle = \left(\sqrt{\tfrac{2}{3}} \;\; -i\sqrt{\tfrac{1}{3}}\right)\begin{pmatrix} \sqrt{\frac{2}{3}} \\ i\sqrt{\frac{1}{3}} \end{pmatrix} = \left(\sqrt{\tfrac{2}{3}}\right)^2 - i\cdot i\left(\sqrt{\tfrac{1}{3}}\right)^2 = \frac{2}{3} + \frac{1}{3} = \underline{\underline{1}} \text{ OK}$$

b) Sannsynlighet for å måle spinnet i $(-x)$-retning:

$$P = |\langle\leftarrow|\psi\rangle|^2$$
$$= \left|\left(\sqrt{\tfrac{1}{2}} \;\; -\sqrt{\tfrac{1}{2}}\right)\begin{pmatrix} \sqrt{\frac{2}{3}} \\ i\sqrt{\frac{1}{3}} \end{pmatrix}\right|^2$$
$$= \left|\sqrt{\tfrac{1}{2}}\sqrt{\tfrac{2}{3}} - i\sqrt{\tfrac{1}{2}}\sqrt{\tfrac{1}{3}}\right|^2$$
$$= \frac{1}{3} + \frac{1}{6}$$
$$= \underline{\underline{\frac{1}{2}}}$$

c) Så har vi gjort en måling og fått kollapset z-komponenten ned på $m_s = -\frac{1}{2}$. Hvis en ny måling av spinn i x-retning etterpå gjøres, er det følgende sannsynlighet for å finne at det peker i $(+x)$-retning:

$$P = |\langle\rightarrow|\downarrow\rangle|^2$$
$$= \left|\frac{1}{\sqrt{2}}(1 \;\; 1)\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right|^2$$
$$= \frac{1}{2}|(1\cdot 0 + 1\cdot 1)|^2$$
$$= \underline{\underline{\frac{1}{2}}}$$