Universal scaling of spontaneous imbibition for arbitrary petrophysical properties: Water-wet and mixed-wet states and Handy's conjecture

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ABSTRACT

Spontaneous imbibition (SI) is a key process in many petrophysical applications, ranging from the mass transfer in fractured reservoirs during a waterflood to wettability characterization of rock samples, or steam migration in geothermal reservoirs. Scaling groups are an essential tool for upscaling laboratory data and modeling and describing SI. A general form has been debated for over 90 years, and several dozen specific groups have been proposed. Here, we give the first general scaling group for arbitrary wettability state, viscosity ratios, rock type, initial water content, and boundary conditions. The result is obtained by extending recent findings for water-wet systems but otherwise arbitrary properties (Schmid and Geiger, 2012) to the mixed-wet case. The group is based on the only known exact, general solution to Darcy's equation with capillarity, and we show that this solution can be viewed as the capillary analogue to the Buckley–Leverett solution for viscous dominated flow. Our group serves as a 'master equation' that contains many of the previously obtained groups as special cases, and its generality can be used to give the first predictive theory for the validity range of specific groups. Based on the universal group, we show that SI is best characterized by the cumulative inflow of the wetting phase and not by the movement of the wetting front, as has been conjectured. Furthermore, our results give strong evidence that Darcy's equation is suitable for describing SI, contrary to what has been hypothesized. The general correlation can be fitted by an exponential model for mass transfer that closely correlates 45 published water–oil, and water–air SI experiments obtained for widely different petrophysical properties.

1. Introduction

Spontaneous imbibition (SI) occurs if a wetting fluid (like water or brine) enters a porous medium, and displaces a non-wetting fluid (like oil, gas or CO_2), driven by capillary forces only. It is a process that is of crucial importance for the evaluation of the wettability of a rock (Jadhunandan and Morrow, 1991; Morrow et al., 1994), and also is the key production mechanism in the world's largest remaining oil reservoirs (Morrow and Mason, 2001). Over 60% of the world's remaining oil reserves are stored in naturally fractured carbonate rocks (Beydoun, 1998). There, the oil is locked in the low permeability rock matrix, surrounded by high permeability fractures, and SI of water is often the only way by which the oil migrates from the rock matrix into the fractures and be produced.

Furthermore, SI is important for the trapping of CO_2 in coal seam and the creation of methane (Chaturvedi et al., 2009), steam migration in geothermal reservoirs (Li and Horne, 2009) and the mechanical stability and distribution of gas hydrates (Anderson et al., 2009; Clennell et al., 1999).

Scaling groups are used to characterize the influence of key parameters on SI, and are essential in any context where SI needs to be understood. For example, they are the bottleneck for an appropriate upscaling of laboratory data (Morrow and Mason, 2001), lie at the heart of modeling and simulating flow in fractured and heterogeneous reservoirs (Barenblatt et al., 1960; Warren and Root, 1963), or are needed as the starting point for evaluating the feasibility of water injection into geothermal reservoirs (Li and Horne, 2009). The enormous practical importance of SI and scaling groups has led to major research activity in that field. In order to resolve how key parameters influence SI and how they should be incorporated into specific scaling groups, a great number of numerical studies on the continuum scale (Behbahani et al., 2006; Delijani and Pishvaie, 2010; Hazlett, 1995; Pooladi-Darvish and Firoozabadi, 2000; Ruth et al., 2000; Standnes, 2006), the pore-scale (Behbahani and Blunt, 2005; Gruener et al., 2012), and the molecular scale (Martic et al.,

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Nomenclature

Greek symbols and units

λ	similarity variable (mt$^{-1/2}$)
λ_α	mobility of phase α (s/Pa)
μ_α	viscosity of phase α (Pa/s)
$\Delta\rho$	density difference between water and oil phase (kg/m^3)
σ	surface tension (N/m)
$\tau(S_w)$	coefficient for dynamic p_c (Pa s)
τ_c	characteristic time (1/s)
ϕ	porosity (–)

Roman symbols and units

A	measurement of a porous medium's, ability to imbibe, Eq. (8) (ms$^{-1/2}$)
A_i	area open to imbibition (m^2)
N_B^{-1}	inverse Bond number (–)
c	proportionality constant (–)
$D(S_w)$	capillary dispersion, Eq. (6) (m^2/s)
f	fractional flow function without p_c, Eq. (6) (–)
f_i	abbreviation for $f(S_{wi})$ (–)
F	fractional flow function with p_c Eq. (11) (–)
g	gravitational constant (m/s^2)
H	height of core (m)
J	Leverett J-function (–)
K	absolute permeability (m^2)
L_c	characteristic length, Eq. (18) (m)
l_{A_i}	distance between A_i and now-flow boundary (m)
p_α	pressure (Pa)

p_d	entry pressure (Pa)
$Q_w(t)$	cumulative 1D volume of wetting phase injected/imbibed (m)
q_t	total velocity (m/s)
R	recovery (%)
R_∞	ultimate recovery (%)
S	saturation (–)
S_w^*	effective saturation (–)
S_{wi}	initial water saturation (–)
S_{BC}	water saturation at left boundary (–)
$S_{\alpha r}$	residual saturation of the phase α (–)
t	time (s)
t_a	ageing time (s)
t^*	time when the analytical solutions stop to be valid (s)
$t_{d,inflow}$	dimensionless time (s)
$t_{d,front}$	dimensionless time (s)
$t_{d,Ma}$	dimensionless time (s)
V_b	the bulk volume of the matrix (m^3)

Subscripts

α	$\alpha \in \{w,o\}$
c	capillary
m	matrix
f	fractures
S	saturation
SI	spontaneous imbibition
T	transversal
w	water
o	oil
WF	water flood

2002), experiments (see Tables 3–8), and analytical solutions for special cases of SI (Table 1) have been proposed (Table 2). Despite this intense research activity, however, and although the research on scaling groups and SI spans more than 90 years (Lucas, 1918; Washburn, 1921), not even apparently simple questions – like the influence of viscosity ratios on SI – have been resolved satisfactorily (Schmid and Geiger, 2012).

In this work, we extend a universal scaling group for water-wet systems (Schmid and Geiger, 2012) to the case of mixed-wet systems, and give the first universal scaling group for spontaneous, counter-current imbibition for arbitrary petrophysical properties. We show the validity of our group by applying it to 55 published SI studies for water–oil and water–air experiments for a wide range of viscosity ratios, initial water content,

Table 1
Previously derived analytical solutions for two-phase flow with capillary effects. To resolve the influence of capillarity, all of them need to employ additional, non-essential assumptions that restrict their applicability. On the contrary, it can be shown (Schmid et al., 2011, Appendix A) that the solution given in McWhorter and Sunada (1990) is general. It can be viewed as the 'Buckley–Leverett Analogue' for counter-current SI (see Section 3.2). This makes the derivation of further specific solutions unnecessary.

Author and year	Assumption
Fokas and Yortsos (1982), Yortsos and Fokas (1983), Philip (1960), Chen (1988), Ruth and Arthur (2011), Wu and Pan (2003)	Specific functional forms for k_{rw}, k_{ro}, p_c
Kashchiev and Firoozabadi (2002)	Steady-state, i.e. $\frac{\partial S_w}{\partial t} = 0$
Li et al. (2003)	Piston-like displacement, i.e. $F(x,t) = \frac{q_w(x^*,t)}{q_w(0,t)}$
Barenblatt et al. (1990), Zimmerman and Bodvarsson (1989), Tavassoli et al. (2005a, b), Mirzaei-Paiaman et al. (2011)	Approximate solution for the weak form
Handy (1960), Chen et al. (1995), Sanchez Bujanos et al. (1998), Rangel-German and Kovscek (2002)	Existence of an equivalent constant capillary diffusion coefficient
Ruth et al. (2007)	Self-similarity behaves according to specific functional form
Cil and Reis (1996), Reis and Cil (1993)	Linear capillary pressure, i.e. $\frac{dp_c}{dx} = \frac{p_c(S_0)}{L}$
Rasmussen and Civan (1998), Civan and Rasmussen (2001)	Asymptotic approximation of Laplace transformation for S_w
Zimmerman and Bodvarsson (1991)	Piecewise linear S_w profile

Table 2

The Lucas–Washburn scaling and some of the recently defined t_d. Characteristic values are denoted by $()^*$, and $\bar{S}(t) = \int_{x_d=0}^{x_d=1} S_{eff} \, dS_{eff}$. It is now apparent, how previous authors (unknowingly) have derived successively better expressions for t_d by giving approximations to the integral in Eq. (3). A specific t_d will give a good scaling if c is the same for the different data sets, and thus c can be used to predict the validity of a special scaling group (Fig. 4(a)).

Author	Dimensionless time	Assumption	Proportionality constant
Lucas–Washburn (1918/21), Washburn (1921)	$t_d \propto \frac{1}{2} \frac{1}{L_c^2} r \frac{\sigma}{\mu_w} t$	$\mu_o \ll \mu_w$	$c \approx \int_{S_0}^{S_{BC}} \frac{(S_w-S_0)k_{rw}J'(S_w)}{F(S_w)} dS_w$
Rapoport (1955), Mattax and Kyte (1962)	$t_d \propto \frac{1}{L_c^2} \sqrt{\frac{K}{\phi \mu_w}} \sigma t$	$\mu_o = \mu_w$	$c = \int_{S_0}^{S_{BC}} \frac{(S_w-S_0)\sqrt{k_{rn}k_{rw}}J'(S_w)}{F(S_w)} dS_w$
Ma et al. (1997)	$t_d \propto \frac{1}{L_c^2} \sqrt{\frac{K}{\phi}} \frac{\sigma}{\sqrt{\mu_w \mu_n}} t$	$\mu_o \approx \mu_w$	$c \approx \int_{S_0}^{S_{BC}} \frac{(S_w-S_0)\sqrt{k_{rn}k_{rw}}J'(S_w)}{F(S_w)} dS_w$
Zhou et al. (2002)	$t_d \propto \frac{1}{L_c^2} \sqrt{\frac{K}{\phi}} \sigma \left(\frac{\lambda_w \lambda_n}{\lambda_t}\right)^* t$	$\left(\frac{\lambda_w \lambda_o}{\lambda_t}\right)(S_w) \approx \left(\frac{\lambda_w \lambda_o}{\lambda_t}\right)^*$	$c \approx \int_{S_0}^{S_{BC}} \frac{(S_w-S_0)J'(S_w)}{F(S_w)} dS_w$
Behbahani and Blunt (2005)	$t_d \propto \frac{1}{L_c^2} \sqrt{\frac{K}{\phi}} \sigma \lambda_w^* t$	$MW: \lambda_w \ll \lambda_o$	$c \approx \int_{S_0}^{S_{BC}} \frac{(S_w-S_0)J'(S_w)}{F(S_w)} dS_w$
Li and Horne (2006)	$t_d \propto \frac{1}{L_c^2} \sqrt{\frac{K}{\phi}} \sigma \left(\frac{\lambda_w \lambda_n}{\lambda_t}\right)^* J^*$ $\cdot (S_{BC}-S_0)t$	$F(x,t) = \frac{q_w(x^*,t)}{q_w(0,t)}$	$c \approx \frac{1}{F(S_w^*)}$
This work	$t_d \propto \left(\frac{2A}{\phi L_c}\right)^2 t$	–	$c = 1$

Table 3

Parameter set used in for the experiments in Zhang et al. (1996). The rock was water-wet Berea sandstone, and $S_0 = 0$ for all experiments. The smallest inverse Bond number is $N_B^{-1} = 145$ for data set BD15, and thus the assumption of negligible gravity is valid.

Core	Boundary cond.	L_c (cm)	k (mD)	ϕ (–)	μ_w (Pa s)	μ_a (Pa s)	Wett. phase	Non-wett. phase	σ (mN/m)	A (m/$\sqrt{s}$)
AA01	AFO	0.5364	510.8	0.218	9.67×10^{-4}	0.03782	Brine	Oil mixture	50.62	7.307×10^{-6}
AA02	AFO	0.8029	498.5	0.219	9.67×10^{-4}	0.03782	Brine	Oil mixture	50.62	7.287×10^{-6}
AA03	AFO	0.9723	519.8	0.222	9.67×10^{-4}	0.03782	Brine	Oil mixture	50.62	7.44×10^{-6}
AA04	AFO	1.089	521.7	0.224	9.67×10^{-4}	0.03782	Brine	Oil mixture	50.62	7.45×10^{-6}
AA05	AFO	1.1837	505.5	0.215	9.67×10^{-4}	0.03782	Brine	Oil mixture	50.62	7.21×10^{-6}
AA06	AFO	1.3059	501.6	0.218	9.67×10^{-4}	0.03782	Brine	Oil mixture	50.62	7.27×10^{-6}
BC21	OEO	6.092	481.9	0.213	9.67×10^{-4}	0.00398	Brine	Soltrol	47.38	1.25×10^{-5}
BC13	OEO	4.998	503.6	0.209	9.67×10^{-4}	0.03782	Brine	Soltrol	47.38	7.05×10^{-6}
BC22	OEO	5.687	496.8	0.208	9.67×10^{-4}	0.1563	Brine	White oil	51.77	4.46×10^{-6}
BD15	TEC	1.3506	523.8	0.214	9.67×10^{-4}	0.00398	Brine	Soltrol	47.38	1.28×10^{-5}
BD14	TEC	1.3506	518.9	0.218	9.67×10^{-4}	0.03782	Brine	Oil mixture	50.62	7.34×10^{-6}
BD18	TEC	1.3506	509.7	0.218	9.67×10^{-4}	0.1563	Brine	White oil	51.77	4.65×10^{-6}
BA31	TEO	3.87	907.1	0.214	9.67×10^{-4}	0.03782	Brine	Oil mixture	50.62	4.65×10^{-6}

Table 4

Parameter set used for the experiments in Hamon and Vidal (1986). They used a synthetic, water-wet porous medium. The smallest inverse Bond number is $N_B^{-1} = 5.7$ for data set A10_85, and thus the assumption of negligible gravity is valid.

Core	Boundary cond.	L_c (cm)	k (mD)	ϕ (–)	μ_w (Pa s)	μ_o (Pa s)	Wett. phase	Non-wett. phase	σ (mN/m)	S_0 (–)	A (m/$\sqrt{s}$)
A10	OEO	9.7	4000	0.472	0.001	0.0115	Water	Refined oil	49.0	0.189	3.36×10^{-5}
A10-20	OEO	19.7	3430	0.453	0.001	0.0115	Water	Refined oil	49.0	0.187	3.1×10^{-5}
A10-30	OEO	30.0	3830	0.453	0.001	0.0115	Water	Refined oil	49.0	0.151	3.18×10^{-5}
A10-40	OEO	40.0	3550	0.478	0.001	0.0115	Water	Refined oil	49.0	0.172	3.33×10^{-5}
A10-85	OEO	84.7	3000	0.478	0.001	0.0115	Water	Refined oil	49.0	0.164	3.13×10^{-5}
A10-VI-20	TEO	19.8	3200	0.456	0.001	0.0115	Water	Refined oil	49.0	0.164	3.17×10^{-5}
A10-X-20	AFO	20.0	2300	0.458	0.001	0.0115	Water	Refined oil	49.0	0.132	2.92×10^{-5}

wettability states, different rock types and different boundary conditions (Tables 3–6, 8).

Our group is derived rigorously from the only known general, exact solution of the two-phase Darcy equation for the case of counter-current imbibition (McWhorter and Sunada, 1990; Schmid et al., 2011). We demonstrate that this solution can be viewed as the capillary analogue to the Buckley–Leverett solution for viscous dominated flow (Buckley and Leverett, 1942), which

renders the derivation of further specific analytical solutions for that case (Table 1) unnecessary. No assumptions other than those needed for Darcy's model are made. No fitting parameters are introduced. All the information present in the two-phase Darcy equation is incorporated into our group. Consequently, our group acts as 'master equation' that contains many of the previously defined scaling groups as special cases (Table 2). Due to the specialized nature of previously defined scaling groups, the

Table 5
Parameter set used for the experiments in Zhou et al. (2002) for cores 2–5, and Bourbiaux and Kalaydjian (1990) for GVB-3 and GVB-4, where a water-wet diatomite and a water-wet Berea sandstone were used, respectively. The smallest inverse Bond number is $N_B^{-1} = 880$ for data set 3, and thus the assumption of negligible gravity is valid.

Core	Boundary cond.	L_c (cm)	k (mD)	ϕ (–)	μ_w (Pa s)	μ_o (Pa s)	Wett. phase	Non-wett. phase	σ (mN/m)	S_0 (–)	A (m/$\sqrt{s}$)
2	OEO	9.5	6.1	0.72	0.001	8.4×10^{-4}	Water	n-decane	51.4	0	3.41×10^{-5}
3	OEO	9.5	7.9	0.77	0.001	2.5×10^{-2}	Water	blandol	45.7	0	1.35×10^{-5}
4	OEO	9.5	2.5	0.78	0.001	8.4×10^{-4}	Water	n-decane	51.4	0	2.72×10^{-5}
5	OEO	9.5	6.0	0.68	0.001	8.4×10^{-4}	Water	n-decane	51.4	0	2.86×10^{-5}
GVB- 3	OEO	29.0	124.0	0.233	0.0012	0.0015	Water	Soltrol-130	35.0	0.4	9.48×10^{-5}
GVB-4	TEO	14.5	118.0	0.233	0.0012	0.0015	Water	Soltrol-130	35.0	0.411	9.67×10^{-5}

Table 6
Parameter set used for the experiments in Fischer et al. (2006) where a water-wet Berea sandstone was used. A brine-glycerol mixture was used as wetting phase, and a Soltrol 220-white oil mixture as non-wetting phase. The obtained viscosity-ratios vary over three orders of magnitude. In all cases $S_0 = 0$. The smallest inverse Bond number is $N_B^{-1} = 161$ for data set EV6_23, and thus the assumption of negligible gravity is valid.

Core	Boundary cond.	L_c (cm)	k (mD)	ϕ (–)	μ_w (Pa s)	μ_o (Pa s)	Wett. phase	Non-wett. phase	σ (mN/m)	A (m/$\sqrt{s}$)
EV6-22	OEO	7.18	109.2	0.18	0.495	0.0039	Brine-glyc.	Soltrol 220-white oil	28.9	4.29×10^{-7}
EV6-18	OEO	7.62	140.0	0.181	0.001	0.063	Brine-glyc.	Soltrol 220-white oil	51.3	3.74×10^{-6}
EV6-21	OEO	7.7	107.3	0.187	0.0278	0.0039	Brine-glyc.	Soltrol 220-white oil	34.3	1.86×10^{-6}
EV6-13	OEO	7.75	113.2	0.187	0.001	0.0039	Brine-glyc.	Soltrol 220-white oil	50.5	7.48×10^{-6}
EV6-14	OEO	7.66	127.2	0.178	0.0041	0.0039	Brine-glyc.	Soltrol 220-white oil	41.2	4.45×10^{-6}
EV6-20	OEO	7.52	132.9	0.181	0.0041	0.0633	Brine-glyc.	Soltrol 220-white oil	41.7	2.45×10^{-6}
EV6-16	OEO	7.78	136.8	0.181	0.0278	0.0633	Brine-glyc.	Soltrol 220-white oil	34.8	1.36×10^{-6}
EV6-23	OEO	7.36	132.1	0.179	0.0977	0.0039	Brine-glyc.	Soltrol 220-white oil	31.3	9.92×10^{-7}
EV6-15	OEO	7.3	107.0	0.183	0.4946	0.0633	Brine-glyc.	Soltrol 220-white oil	29.8	3.93×10^{-7}
EV6-17	OEO	7.54	128.1	0.19	0.0977	0.0633	Brine-glyc.	Soltrol 220-white oil	32.1	8.59×10^{-7}

question of the range of validity was often left wide open (Schmid and Geiger, 2012), and we demonstrate how the generality of our approach allows the prediction of the validity range of specialized groups by the derivation of a proportionality constant c (Table 2). This is the first predictive theory for evaluating scaling groups.

Furthermore, we show that Handy's conjecture is wrong. In an attempt to link scaling groups to some physical property of the SI process, Handy (1960) and subsequent authors (Babadagli and Zeidani, 2004; Cil et al., 1998; Li et al., 2002; Li and Horne, 2009; Schembre et al., 1998) speculated that SI can be characterized by the frontal movement of the wetting phase. We show rigorously that although a scaling based on the frontal movement gives better results than some previously defined methods, SI systems are best characterized by the total volume of the wetting phase imbibed.

Besides resolving these crucial 'practical' questions on how key petrophysical properties affect SI, our scaling groups serve as a theoretical tool to assess the validity of Darcy's equation for describing SI. During recent years the theoretical framework itself on how to model SI has been debated. Both in the oil-community (Barenblatt et al., 2003), and in related areas such as hydrology, physics and engineering (Hall, 2007; Mirzaei and Das, 2007), it has been proposed that the extended Darcy model is not suitable for describing SI. Instead a Darcy formulation should be used that additionally contains parameters determined from thermodynamic considerations which describe capillary pressure as a function of both saturation and rate of change (Barenblatt et al., 2003; Mirzaei and Das, 2007, for a recent overview see Hall, 2007). Since our group is based on the validity of the standard Darcy equation for two-phase flow, the ability of our group to correlate experimental data can be used as a benchmark of whether the standard Darcy formulation for multiphase flow is suitable to describe SI. Our results strongly suggest that accounting for the capillary pressure dependency on saturation only is

sufficient, and that including additional dependencies for capillary pressure is unnecessary.

Finally, we show that an exponential model for mass transfer closely correlates experimental data independent of petrophysical properties. This yields the first general expression for mass transfer due to SI.

The remainder of this paper is structured as follows: In Section 2 we review known scaling groups, and related open questions. In Section 3 we introduce the mathematical formulation and notation. The lack of knowledge of an exact analytical solution for counter-current SI has long been the bottleneck for the derivation of a general scaling group and many other applications. We repeat a known exact solution for SI (McWhorter and Sunada, 1990; Schmid et al., 2011). The generality of this solution has long been overlooked (Schmid et al., 2011), and many authors (Table 1) tried to derive specialized solutions. We demonstrate that it can be viewed as the capillary analogue to the Buckley–Leverett equation for viscous dominated flow (Buckley and Leverett, 1942), which renders the derivation of further specialized solutions (Table 1) unnecessary. While the analytical solutions have been derived for a semi-infinite medium, we show that they are valid in any finite medium up to some time t^*. In Section 4 we introduce two scaling groups that comprise all the information present in the two-phase Darcy formulation: The first one, $t_{d,front}$ is based on the frontal movement of the wetting phase, and the second one, $t_{d,inflow}$ is based on the total volume of wetting phase imbibed. We correlate 45 published SI experiments with both groups, and show that while $t_{d,front}$ already strongly improves previously proposed groups, it is $t_{d,inflow}$ that gives the best results, contrary to what has been conjectured (Handy, 1960). Since $t_{d,inflow}$ has a general form, this strongly indicates that the standard Darcy model is suitable for describing SI, in contrast to what has been speculated (Barenblatt et al., 2003; Hall, 2007; Mirzaei and Das, 2007). In Section 5 we extend the results to the

case of mixed-wet systems, and in Section 6 we show how this can be applied to obtain a mass-transfer function for dual-porosity models that is valid for arbitrary petrophysical properties and therefore overcomes all the major limitations of previously proposed transfer functions for SI. We finish the paper with some conclusions. A list of symbols is given in the Nomenclature.

2. Previously proposed scaling groups and open questions

2.1. Introduction

Scaling groups for SI were derived mainly by two ways. Either a curve was fitted against a large body of experimental data where only one parameter (like the characteristic length) was varied (e.g. Zhang et al., 1996), or simplifying theoretical assumptions in the form of scaling laws (Mattax and Kyte, 1962; Rapoport, 1955) or other non-essential assumptions (Table 1) were employed. Both approaches yield scaling groups whose applicability is strongly restricted. On the one hand, a general theoretical understanding on why a certain group works and when it would fail were often left unanswered. On the other hand, the incorporation of three aspects into scaling groups – namely the influence of viscosity ratios, the influence of initial water content, and the influence of wettability effects – remain open which play a central role in practical applications.

2.2. Viscosity ratios

The viscosities of the phases act as a weighting factor for the relative permeabilities, and it is unclear how this weighting should look like depending on the viscosity ratio. It has long been known (Lucas, 1918; Washburn, 1921) that for a negligible non-wetting phase viscosity μ_o, SI is proportional to $1/\sqrt{\mu_w}$, where μ_w is the wetting phase viscosity (see the Nomenclature). Motivated by this result, a scaling group for two-phase flow was proposed that assumed a scaling proportional to $1/\sqrt{\mu_w \mu_o}$ (Ma et al., 1997) where μ_o is the viscosity of the non-wetting phase. However, a great number of experimental studies (Behbahani and Blunt, 2005; Fischer and Morrow, 2006; Fischer et al., 2006) and numerical results (Behbahani and Blunt, 2005) showed that for a viscid non-wetting phase, a scaling with the geometric mean in general does not hold. Although subsequently many attempts have been made to generalize predictions for arbitrary viscosity ratios (Fischer and Morrow, 2006; Fischer et al., 2006; Høgnesen et al., 2004; Mason et al., 2010; Reis and Cil, 1993; Ruth et al., 2004; Standnes, 2010a,b), a scaling group for arbitrary viscosity ratios, based on rigorous physical considerations, remains unknown. Similarly, the question on how a single relative permeability value should be chosen in dependence of the viscosity ratio and such that it characterizes the strong non-linear dependence on the wetting saturation over the whole saturation range remains open (Mason et al., 2010; Morrow and Mason, 2001).

2.3. Initial water saturation

The influence of initial water saturation S_{wi} is twofold: On the one side, if a rock is aged at a certain S_{wi}, the aging time, the rock and the oil properties will influence the capillary-hydraulic properties depending on S_{wi} (Graue et al., 1999; Xie and Morrow, 2001; Zhou et al., 2000). This is discussed in the next subsection. On the other hand, if a rock is not aged, S_{wi} does not change the basic petrophysical properties (Narahara et al., 1993), and can be at either residual water saturation S_{wr} or above. If $S_{wi} > S_{wr}$, then competition occurs between the low capillary pressure force and the high phase mobilities (Lee and Karpyn, 2012).

Also, since the establishment of an $S_{wi} > 0$ can be difficult, many SI experiments are performed for the case where the core is fully saturated with oil or air (Zhang et al., 1996; Zhou et al., 2002), i.e. $S_{wr} = 0$, while one would like to understand SI behavior under actual reservoir conditions where $S_{wi} > 0$. So far, both the effect of competition and the effect of different S_{wr}'s have only been characterized for cases where the ratio of non-wetting to wetting phase viscosity is close to one, and if the capillary pressure and the wetting behavior can be characterized by a single value (Li and Horne, 2006), which is unlikely in realistic porous media (Valvatne and Blunt, 2004; Jackson et al., 2003); it also blurs the difference between different rock types, i.e. different capillary-hydraulic properties.

2.4. Wettability effects and mixed-wet systems

Capillary pressure curves and the phase mobilities not only depend on the fluids, but also on the geometry of the pore structure (Jackson et al., 2003; Valvatne and Blunt, 2004). Consequently, even for the water-wet case, they are different
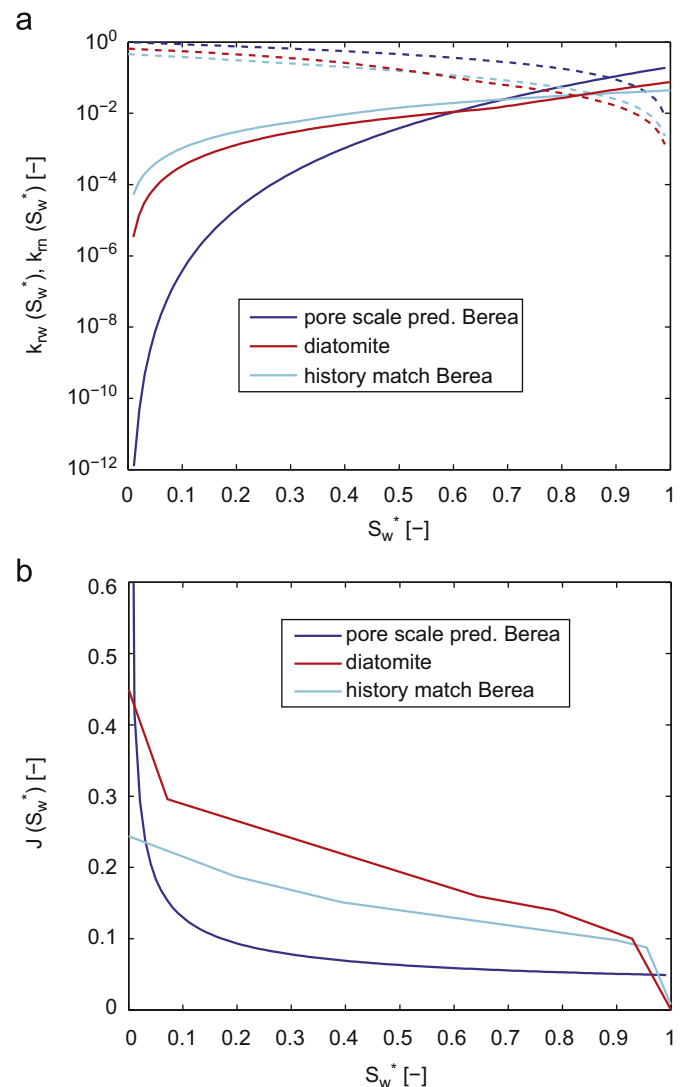

Fig. 1. Capillary-hydraulic properties vs. normalized saturation for the water-wet experiments (Tables 3–5) (a) relative permeabilities for the wetting (−) and non-wetting phase (− −), and (b) dimensionless J-function. Pore-scale predictions (Valvatne and Blunt, 2004) for the Berea sandstone and the synthetic porous material, and a history match that assumes a Darcy model for sandstone (Bourbiaux and Kalaydjian, 1990), and a non-Darcy model for the diatomite (Schembre and Kovscek, 2006) were used.

for different materials (Fig. 1). Up to now, however, scaling groups try to characterize the influence of capillary pressure and wetting by some single value that is representative of the entire porous medium (Benavente et al., 2002; Li and Horne, 2006; Marmur, 2003; Pordel Shahri et al., 2012; Tavassoli et al., 2005a; Zhou et al., 2002).

Even for the same rock type, however, this approach is unsatisfactory if different wettabilities need to be compared. Understanding different wettability states is of major importance since many oil reservoirs are not water-wet, but mixed-wet (Rezaveisi et al., 2012). If a rock sample is aged at different initial water saturations, S_{wi} controls how many pores are exposed to the non-wetting phase, and thus S_{wi} controls how many pores can change their wettability by the adsorption of the non-wetting phase (Graue et al., 1999; Xie and Morrow, 2001; Zhou et al., 2000). At the end of the aging process, the rock contains both water-wet (WW), and oil-wet (OW) pores, known as 'mixed-wet' (MW) behavior (Salathiel, 1973). MW systems are of enormous practical importance since the majority of the oil reservoirs are not WW, but MW (Anderson, 1987a,b). SI experiments for MW systems show differences in recovery times that can be several orders of magnitude compared to WW experiments (Graue et al., 1999; Xie and Morrow, 2001; Zhou et al., 2000), specific relative permeabilities and p_c curves (Fig. 6(a), (b)), and are also characterized by two different residual oil saturations (Fig. 7). Although several attempts have been made to develop scaling groups for a MW scenario, most of them try to lump these different effects – differences in recovery times, different S_{or}'s, particular relative permeabilities – into a single 'wettability factor' (Cil et al., 1998; Gupta and Civan, 1994; Lavi et al., 2008; Xie and Morrow, 2001; Zhou et al., 2002). This factor is derived from a mere empirical fit to experiments, lacks any physical meaning, does not allow any theoretical insights and has only limited predictive powers (Anderson, 1987a; Behbahani and Blunt, 2005; Marmur, 2003).

2.5. Can Darcy's model describe SI?

In addition to these three practical issues discussed in the previous sections, the theoretical framework itself for describing SI has recently become the center of a debate in physics and engineering. While the single-phase Darcy model can be derived rigorously from first principles (Hassanizadeh, 1986), this is not the case for the multiphase extension of Darcy's equation where capillary pressure only depends on saturation (Hassanizadeh and Gray, 1993; Muskat, 1949), and it has been proposed that it should be replaced by a model that considers additional dependencies of capillary pressure on the rate of saturation change (Barenblatt et al., 2003), for recent reviews see Hall (2007) and O'Carroll et al. (2010). A number of formulations have been proposed (Hall, 2007; O'Carroll et al., 2010) that introduce additional parameters into the classical Darcy formulation. But it is unclear, if and under which conditions which parameters should additionally be taken into account.

2.6. The Buckley–Leverett analogue, universal scaling group and the validity of Darcy's equation

In the following, we extend recent results (Schmid and Geiger, 2012) for a scaling group for WW conditions but otherwise arbitrary petrophysical properties to the MW case and for arbitrary petrophysical properties. Therefore, for the first time, all three aforementioned practical aspects will be accounted for. We furthermore show that Handy's conjecture (Handy, 1960) is wrong. Our scaling group is derived rigorously from an exact solution of the extended Darcy equation. We do not make any

assumptions other than those needed for Darcy's model. We do not introduce any fitting parameters. The exact solution can be viewed as analogue to the classical Buckley–Leverett solution for two-phase flow in the viscous limit (Buckley and Leverett, 1942), and thus should finalize the long search for a capillary counterpart to Buckley–Leverett's equation for viscous dominated flow (Table 1). We show that our group is a 'master equation' for scaling SI, that contains many of the previously defined groups as special cases, and demonstrate how the generality of our approach allows the prediction of the validity range of specialized groups (Table 2). Furthermore, we will give strong evidence that the classical Darcy description for SI is appropriate.

3. Problem formulation, assumptions, and an exact solution for counter-current SI

3.1. Problem formulation

Conservation of mass for two immiscible, incompressible phases at constant temperature through a rigid, homogeneous, horizontal (i.e. gravity can be neglected), one-dimensional porous medium leads to the following equation (Bear, 1972):

$$\phi \frac{\partial S_w}{\partial t} = -\frac{\partial}{\partial t}(q_w),$$
$$S_w + S_o = 1,$$
(1)

where S_w is the water phase saturation, S_o is the non-aqueous phase saturation, and ϕ is the porosity. We assume that the volume flux of the aqueous and non-aqueous phase, q_w and q_o, can be described by the extended Darcy equation (Muskat, 1949), which describes the volume flux due to a gradient in the phase pressures p_w and p_o:

$$q_w = -K \frac{k_{rw}}{\mu_w} \nabla p_w,$$
$$q_o = -K \frac{k_{ro}}{\mu_o} \nabla p_o.$$
(2)

Here K is the absolute permeability, μ_w is the viscosity of the water or aqueous phase, μ_o is the viscosity of the oil (or air), and k_{ro} and k_{rw} are the relative permeability of the oleic and the aqueous phase, respectively. The relative permeabilities describe the impairment of the one fluid phase by the other. The two-phase pressures p_w and p_o are related through the capillary pressure $p_c = p_o - p_w$. Combining Eqs. (1) and (2), we obtain (Bear, 1972)

$$\phi \frac{\partial S_w}{\partial t} = -q_t \frac{df_w}{dS_w} \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x} \left(D(S_w) \frac{\partial S_w}{\partial x} \right),$$
(3)

where $q_t = q_w + q_o$. One can rewrite q_w as an expression of the total volume flux (McWhorter and Sunada, 1990)

$$q_w = f(S_w)q_t - D(S_w) \frac{\partial S_w}{\partial x}.$$
(4)

We consider the boundary and initial conditions

$$S_w(x = 0, t) = S_{BC},$$
$$S_w(x, t = 0) = S_w(\infty, t) = S_0.$$
(5)

The functions $f(S_w)$ and $D(S_w)$ are defined as

$$f(S_w) = \left(1 + \frac{k_o \mu_w}{k_w \mu_o} \right)^{-1},$$
$$D(S_w) = -K \frac{k_o f}{\mu_o} \frac{dp_c}{dS_w}.$$
(6)

Here, f is the fractional flow function without the influence of capillary pressure, and $D(S_w)$ can be thought of as a 'capillary dispersion coefficient' of the fluid phases.

In counter-current SI, the two phases flow into opposite directions, i.e. $q_w = -q_o$, which reduces Eq. (3) to the non-linear dispersion equation:

$$\phi \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x} \left(D(S_w) \frac{\partial S_w}{\partial x} \right). \tag{7}$$

In the following we will only investigate counter-current SI, and for simplicity denote it as SI. In Eqs. (1)–(7), we assumed that k_{rw}, k_{rn} and p_c are unique functions of saturation. Since history effects only occur during a whole imbibition-drainage cycle (Bear, 1972; Valvatne and Blunt, 2004), this assumption is justified. However, even for this case, it has been suggested that the formulation given above is not suitable for describing SI, since one would expect that the fluid phases need some 'relaxation time' to achieve capillary equilibrium. This would introduce an additional dependence of p_c on the rate of change in saturation $\partial S_w / \partial t$ (Barenblatt et al., 2003; Mirzaei and Das, 2007), for a recent review see Hall (2007). We will base all the following analysis on the standard model as given by Eqs. (1)–(7), and will show that a scaling group that incorporates all the information comprised in the standard formulation is capable of characterizing SI. This strongly indicates that a non-standard model is unnecessary to describe SI.

3.2. Exact analytical solution for imbibition: the 'Buckley–Leverett' analogue

If an analytical solution to Eq. (7) is known, a general scaling group can immediately be derived from these analytical expressions. In this section, we therefore shortly review known analytical solutions for capillary dominated two-phase flow, and describe an exact analytical solution to Eq. (7) together with (5). The exact solution is solely based on the assumptions made in Section 3.1. It can be viewed (Schmid et al., 2011, 2012) as the 'capillary analogue' to the Buckley–Leverett solution for viscous dominated flow (Buckley and Leverett, 1942).

While an analytical solution to Eq. (3) for uni-directional viscous dominated flow has long been known (Buckley and Leverett, 1942), the counterpart for capillary-dominated flow has been missing, and the derivation of solutions for capillary dominated two-phase flow has been the subject of ongoing intensive research over the last decades (Table 1). The reasons for this strong interest are clear: Besides the theoretical insights analytical solutions yield, they are important for benchmarking numerical code, they form the basis for scaling groups and they are used to derive transfer functions essential for predicting one of the most important production mechanism in fractured reservoirs. The obtained solutions fall into two categories: In the first category, additional assumptions on Eq. (7) are made, like specific functional forms of $D(S_w)$, that flow occurs under steady-state, and so forth (Table 1). In the second category (McWhorter and Sunada, 1990), no additional assumptions on the physics or $D(S_w)$ are made. Instead, McWhorter and Sunada (1990) impose an additional boundary condition that specifies the inflow as $q_0 = q_w(x = 0,t) = At^{-1/2}$.

In the following, we first will repeat the main results of the solution derived in McWhorter and Sunada (1990). Then we will analyze why the boundary condition on q_0 is redundant and derive an expression for the time t^* when the solutions cease to be valid. Finally, we will demonstrate that the obtained solutions can be viewed as the capillary analogue to the Buckley–Leverett solutions.

McWhorter and Sunada (1990) use that solutions to equations of the dispersion type show self-similar behavior according to $S_w \propto x\sqrt{t}$, i.e. S_w can be written in terms of the self-similar variable $\lambda = xt^{-1/2}$ as $S_w = S_w(\lambda)$ (McWhorter and Sunada, 1990).

A is a parameter that cannot be chosen freely, but depends on the characteristics of the fluid-rock system according to (McWhorter and Sunada, 1990)

$$A^2 = \frac{\phi}{2} \int_{S_0}^{S_{BC}} \frac{(S_w - S_0)D(S_w)}{F(S_w)} \, dS_w, \tag{8}$$

and is related to the cumulative water imbibed by

$$Q_w(t) = \int_0^t q_w(0,t) \, dt = 2At^{1/2}. \tag{9}$$

$F(S_w)$ is the fractional flow function for counter-current SI, i.e. it can be viewed as the 'capillary counterpart' to $f(S_w)$,

$$F(x,t) = \frac{q_w(x,t)}{q_0}, \tag{10}$$

and can be described explicitly by the non-linear equation

$$F(S_w) = 1 - \left(\int_{S_w}^{S_{BC}} \frac{(\beta - S_w)D(\beta)}{F} \, d\beta \right) \cdot \left(\int_{S_0}^{S_{BC}} \frac{(S_w - S_0)D(S_w)}{F} \, dS_w \right)^{-1}. \tag{11}$$

Together with A and its derivative F', the analytical solution for Eq. (7) with (5) and the condition on $q_w(0,t)$ can then be written as (McWhorter and Sunada, 1990)

$$x(S_w,t) = \frac{2A}{\phi} F'(S_w)t^{1/2} = \frac{Q_w(t)}{\phi} F'(S_w). \tag{12}$$

Similarly, an expression for the flux can be obtained as

$$q_w(S_w) = q_0 F'(S_w). \tag{13}$$

The inflow condition $q_w(0,t) = At^{-1/2}$ is the one McWhorter and Sunada (1990) and subsequent authors worked with, so that at first the solution (12) seems like just another specific one.

However, it can easily be shown (Schmid et al., 2011, Appendix A) that $A = -D(S_{BC})dS_w/d\lambda |_{\lambda=0}$. Thus, A is exactly such that $q_w(0,t) = -D(S_0)\partial S_w/\partial x |_{x=0}$. This follows from Eq. (4)

$$q_w(x = 0,t) = -D(S_0)\frac{\partial S_w}{\partial x} = -D(S_0)\frac{dS_w}{d\lambda}\bigg|_{\lambda=0}\frac{\partial \lambda}{\partial x}$$

$$= -D(S_0)\frac{dS_w}{d\lambda}\bigg|_{\lambda=0} t^{-1/2} = At^{-1/2}. \tag{14}$$

Consequently, A does not describe *forced* imbibition, but rather is such that the inflow $q_w(0,t)$ occurs *spontaneously* into the porous medium because of the saturation gradients and the resulting gradients in capillary pressure only. Thus, the boundary condition on $q_w(0,t)$ is redundant, and Eq. (12) describes the standard situation found in the laboratory. Further derivations of analytical solutions (Table 1) for the counter-current case seem unnecessary.

Although the solutions have been derived for an infinite medium according to the initial condition (5), the x-profile for any time $0 \le t < \infty$ has a finite extend (Fig. 2). Thus, the solution is valid in a finite matrix block of length L as long as the wetting front has not reached the end of the block. The time t^* when the solutions stop to be valid in a finite matrix block can hence be obtained from setting $x(S_{wi},t^*) = L$ which yields

$$t^* = \left(\frac{L\phi}{2AF'(S_i)} \right)^2. \tag{15}$$

For any $0 < t < t^*$ the profiles are given by (12), and Q_w is given by (9). In Fig. 4 we show the analytical solution for Q_w vs. the dimensionless time defined below for the data set where the sample-specific capillary-hydraulic properties are known (□ in Fig. 4), data set GVB-3 in Table 5. For early times, the analytical prediction is in good agreement with the data, but for late times it fails to predict the slow down in recovery. This is because for
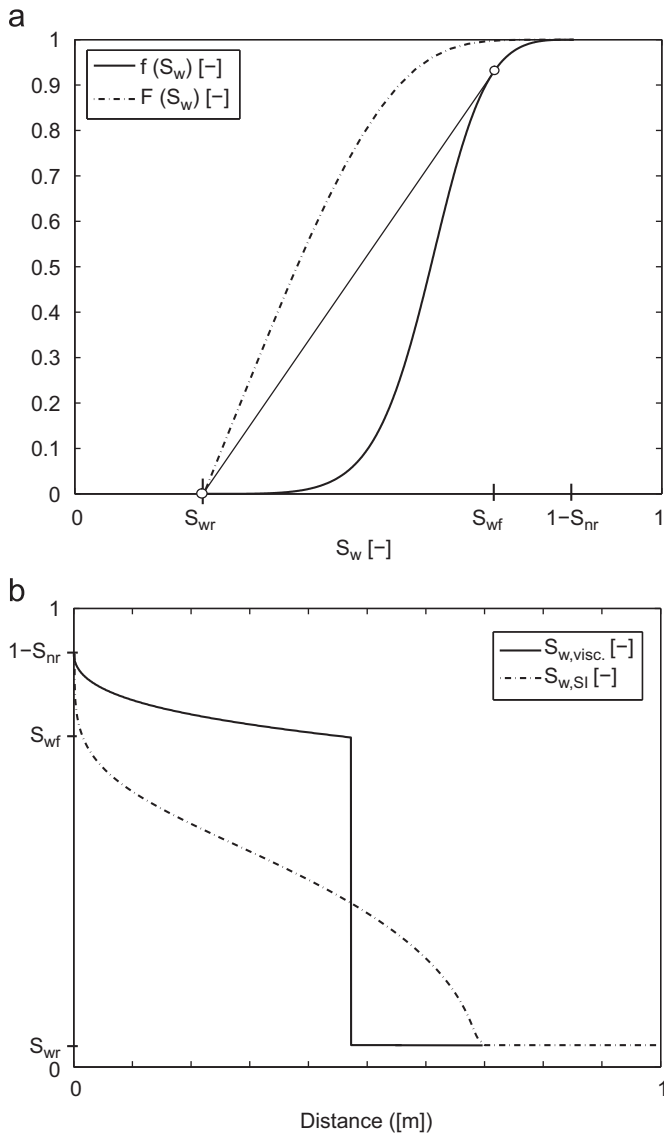
Fig. 2. (a) Fractional flow function vs. saturation and (b) the resulting saturation profiles vs. distance for counter-current SI and viscous dominated, unidirectional flow. The fractional flow function f for the viscous dominated case is s-shaped which according to Eq. (17) leads to a shock in the saturation profiles at $S_w = S_{wf}$ as determined by the Welge tangent (Welge, 1952). F can be viewed as the capillary counterpart to f for counter-current SI, and is concave. This results in a smooth saturation profile according to Eq. (12).

$t > t^*$, the end of the block influences the saturation profile, and Eqs. (12) and (9) are no longer valid.

Eq. (12) shows a remarkable similarity to the classical Buckley–Leverett solution (Buckley and Leverett, 1942) for uni-directional, viscous dominated flow with a constant inflow rate. It is repeated here for convenience: The flux q_w satisfies

$$q_w(S_w) = q_t f'(S_w), \tag{16}$$

and the x-profile is given by

$$x_{viscous}(S_w,t) = \frac{Q_w(t)}{\phi} f'(S_w). \tag{17}$$

Thus, the solutions for $x(S_w,t)$ and q_w in Eqs. (12) and (13) can be viewed as the 'capillary analogue' to the Buckley–Leverett solutions (16) and (17), respectively. Contrary to the viscous fractional flow function, F is concave since $F''(S_w) < 0$, independent of the parametrization used for k_{rw}, k_{ro} and p_c (McWhorter and Sunada, 1990). Hence, F does not show the s-shape that is typical for f (Fig. 2). This reflects the fact that the saturation profile for the capillary dominated case cannot develop a shock-front as the one for the Buckley–Leverett solution, but rather shows a smooth 'capillary belly' profile. Indeed, if one naively tried to draw a Welge tangent (Welge, 1952) to F in order to obtain S_{wf} for the case where capillarity is not ignored, the concave shape of F would yield $S_{wf} = S_0$, i.e. no shock front occurs. Mathematically, this is not surprising, as the conservation equation (7) is parabolic, and thus all its solutions are smooth (Fig. 2).

4. Defining a universal scaling group for SI: cumulative inflow vs. frontal movement

4.1. Introduction

Eq. (12) can be used to introduce a scaling group that incorporates all the information present in the two-phase Darcy equation. There are two possibilities: The scaling group can be based on the frontal movement of the wetting phase, or on the cumulative inflow $Q_w(t)$. Handy (1960) and subsequent authors (Babadagli and Zeidani, 2004; Cil et al., 1998; Li et al., 2002; Schembre et al., 1998) conjectured that it is the frontal movement that characterizes imbibition. Since we can resolve the non-linear equations exactly, we can rigorously show that although this scaling already gives correlations superior to some of the previously defined scaling groups, SI is characterized best by the cumulative inflow.
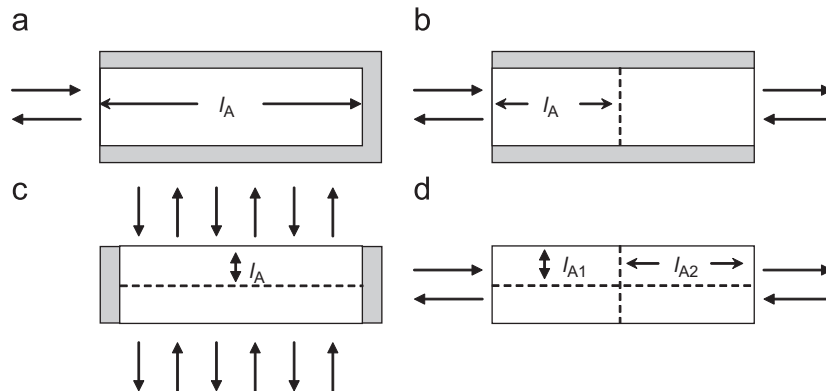

Fig. 3. Counter-current SI in cores with different boundary conditions: (a) one end open (OEO), (b) two-ends open (TEO), (c) two ends closed (TEC), and (d) all faces open (AFO).

To derive a scaling group from Eq. (12), we first normalize x by the characteristic length L_c through x/L_c where (Ma et al., 1997)

$$L_c = \sqrt{\frac{V_b}{\sum_{i=1}^{n} A_i/l_{A_i}}}. \tag{18}$$

V_b is the bulk volume of the matrix, A_i the area open to imbibition with respect to the ith direction, and l_{A_i} is the distance that the imbibition front travels from the imbibition face to the no-flow boundary (Fig. 3). L_c thus has the physical interpretation of quantifying the length a wetting front can travel without meeting a boundary or another imbibition front (Ma et al., 1997). While the question of how to incorporate the influence of viscosity ratios, wettability information, and so forth into scaling groups is open, the correct incorporation of different boundary conditions (Fig. 3) with the help of L_c has been confirmed (Ma et al., 1997; Zhang et al., 1996). For cases where A_i is not constant in time, but grows because for example of slowly filling fractures, this time-dependency needs to be included in L_c (Rangel-German and Kovscek, 2002, 2006). However, this dependency can be treated independently of petrophysical properties, and therefore we will not discuss it here.

In the following, we first will introduce two new scaling groups – one based on the frontal movement of the wetting front and one based on the cumulative inflow of the wetting phase – and then compare them to the commonly used group by Ma et al. (1997) for the data set shown in Tables 3–6. Then we will demonstrate how the generality of our approach can be used to predict the range of validity of specialized groups, and how the generality the scaling group based on Q_w can be used to measure the validity of Darcy's equation.

4.2. Scaling groups

We first introduce the scaling group $t_{d,front}$ that is based on the frontal movement. The position of the front follows from Eq. (12) with $S_w = S_0$, and motivates the introduction of $t_{d,front}$ as

$$t_{d,front} = \left(\frac{x_{front}(t)}{L_c}\right)^2 = \left(\frac{2A}{\phi L_c}F'(S_0)\right)^2 t. \tag{19}$$

If in contrast the normalized cumulative inflow $Q_w(t)/(\phi L_c)$ is used, then we obtain

$$t_{d,inflow} = \left(\frac{Q_w(t)}{\phi L_c}\right)^2 = \left(\frac{2A}{\phi L_c}\right)^2 t$$
$$= \tau_c t. \tag{20}$$

Two remarks are in order. First, note that the above scaling groups try to predict the influence on recovery over time if all the parameters are known. This approach is fundamentally different from dimensionless groups that try to predict parameters like S_{or} from dimensionless groups (e.g. Anton and Hilfer, 1999).

Second, the scaling group based on normalized volume conceptually is the same as the one used in enhanced oil recovery for viscous dominated flow (e.g. Lake, 1989, Chapter 5). The difference of course lies in the fact that we consider capillary driven flow. This results in a $\sqrt{t}$-dependency of the inflow, and requires that $Q_w(t)$ be squared contrary to the t-dependency for viscous dominated floods where $Q_w(t)$ is not squared.

We next show that $t_{d,inflow}$ is suited best to describe SI by correlating 42 published experiments, and comparing $t_{d,front}$, $t_{d,inflow}$ and the often used group by Ma et al. (1997). This comparison shows that $t_{d,inflow}$ is the general scaling group for SI, and forms the so far missing center piece for upscaling, modeling and simulating diverse systems, where SI plays a role. Thus, τ_c can be thought of as a characteristic time that quantifies both the influence of the capillary-hydraulic properties and the

characteristic physical dimensions. Also, $t_{d,inflow}$ can be used as a theoretical tool to assess the validity of Darcy's equation.

4.3. Validity of scaling groups

To demonstrate the validity of our scaling groups, we correlated them with 42 published imbibition studies for water-wet conditions. For the correlation of the different degrees of mixed-wet systems, see Section 5. The experiments are for different boundary conditions (Fig. 3) and lengths L_c (Tables 3–6), and cover all the critical points mentioned in Section 2, i.e. a wide range of viscosity ratios ($0.008 \leq \mu_n/\mu_w \leq 64$) with the special case of the non-aqueous viscosity tending towards zero ($\mu_{air} = 1.8 \times 10^{-5}$ Pa s), varying initial water saturation ($0\% \leq S_0 \leq 40\%$), and different porous materials, i.e. different capillary-hydraulic properties (Berea Sandstone, a synthetic material and diatomite, Fig. 1), see Tables 3–6.

Figs. 4(b) and 5(b) show the correlation of the data with $t_{d,inflow}$ and $t_{d,front}$. For comparison, Figs. 4(a) and 5(a) show the
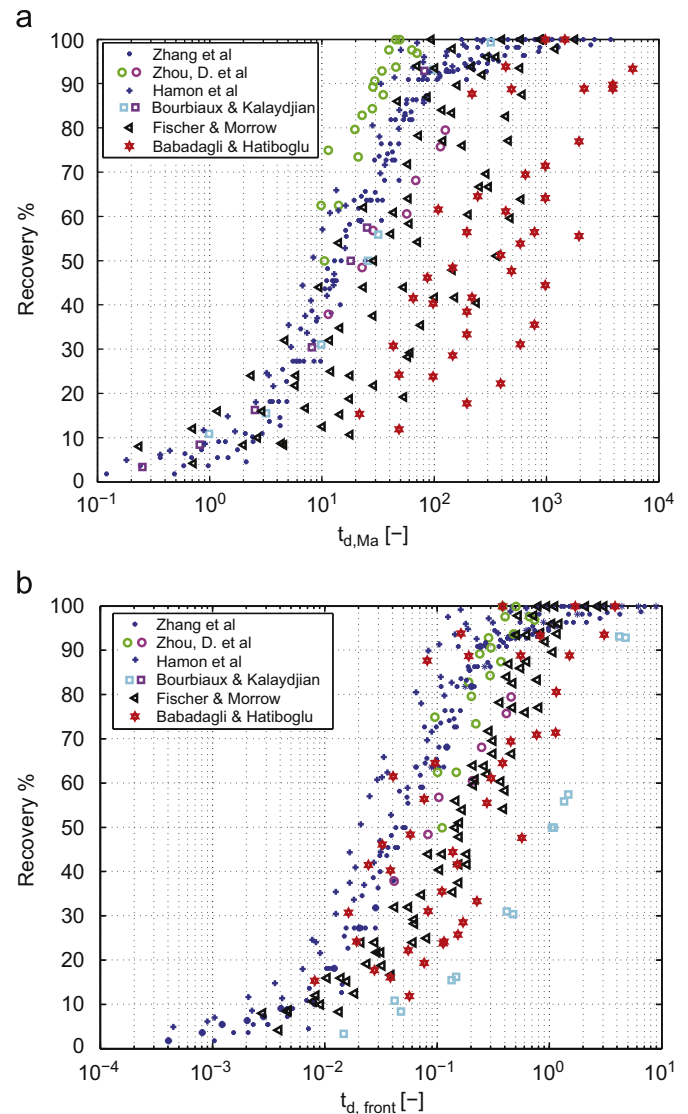
Fig. 4. Recovery of the displaced fluid vs. dimensionless time. (a) Time scaled according to Ma et al. (1997). The scaling does not result in the collapse of the data onto a single curve. The scaling group can only result in a good correlation if the proportionality constant c (Table 2) is the same for all the data sets. (b) Time scaled with $t_{d,front}$. This scaling results in an improved scaling compared to the one shown in (a).
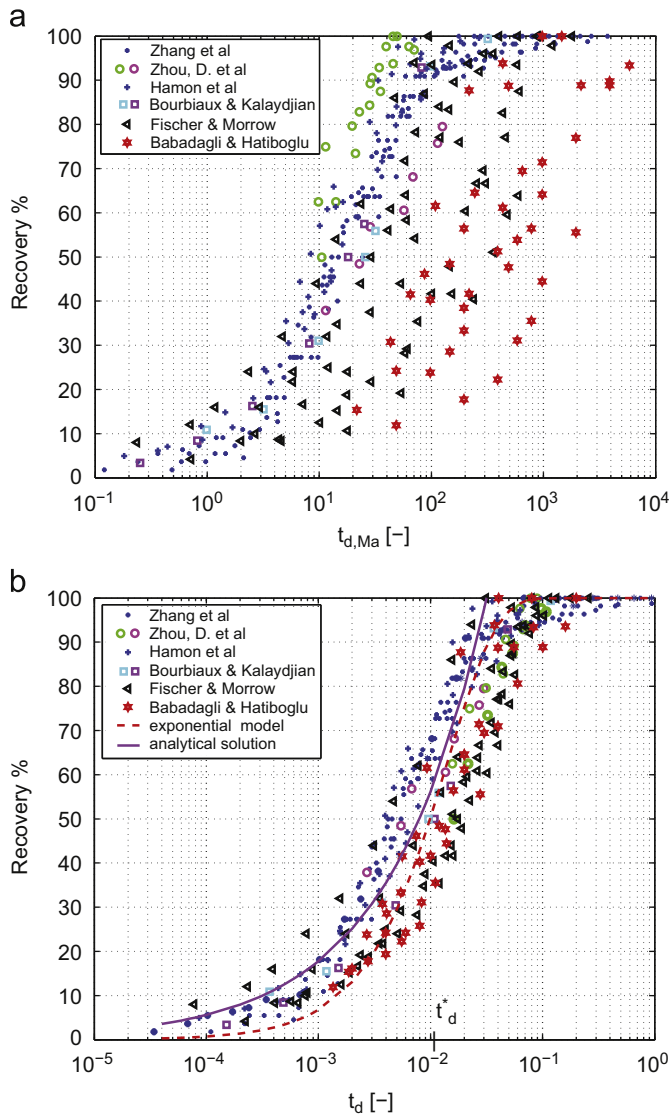
Fig. 5. Recovery of the displaced fluid vs. dimensionless time. (a) Time scaled according to Ma et al. (1997) (repeated from Fig. 5(a)). The scaling does not result in the collapse of the data onto a single curve. (b) Time scaled with $t_{d,inflow}$. This scaling results into a curve with little scatter of the data that strongly reduces both the horizontal and the vertical scatter compared to $t_{d,Ma}$ and $t_{d,front}$ (b). This indicates that SI is best described by the cumulative inflow of the wetting phase rather than the frontal movement. Since $t_{d,inflow}$ comprises all the data present in the Darcy formulation this also indicates that the Darcy model is suitable for modeling SI. The analytical solution is valid as long as $t_d \leq t_d^* = \tau_c t^*$. To correlate the behavior for the whole time range, an exponential model (Aronofsky et al., 1958) is used.

correlation of the same data set with the commonly used scaling group $t_{d,Ma}$ proposed by Ma et al. (1997) (Table 2). Both newly proposed scaling groups yield significantly better results compared to $t_{d,Ma}$. The correlation achieved by $t_{d,inflow}$ gives the best results and indicates that SI is characterized best by the cumulative inflow and not by the frontal movement that previous authors assumed as a starting point for deriving specialized scaling groups (Babadagli and Zeidani, 2004; Cil et al., 1998; Handy, 1960; Li et al., 2002; Schembre et al., 1998). In the remaining parts of this paper we therefore will only discuss $t_{d,inflow}$, and will refer to it as t_d for simplicity. The scaling with t_d strongly reduces the horizontal scatter (i.e. the one for a fixed recovery rate) from a factor of greater than 100 down to approximately 5, and the maximal vertical scatter (i.e. the one for a fixed t_d) from approximately 0.8 to 0.3.

To further improve the scaling, one should use the capillary-hydraulic properties for the specific sample when calculating t_d. Most data sets available in the literature only report SI measurements or (k_{rw}, k_{rn}) relationships, or p_c curves. In order to calculate t_d we therefore assumed that the (k_{rw}, k_{rn}, p_c) sets measured for a certain rock type are representative for a given material, see Section 4.4.

It is also interesting to note that the data in Fig. 4(b) scatters around the curve given by the analytical solution for $Q_w(t)$ that has been calculated for data where the sample-specific capillary-hydraulic properties are known. It is not clear, whether this is true only for particular data sets, or shows that the analytical solution is a 'master curve' for early times on which all data would collapse if better predictions for the capillary-hydraulic properties were available. One would have to calculate the analytical prediction for more data sets where the sample specific properties are known. As we explained, however, complete data sets are rare.

4.4. The standard Darcy equation is suitable for modeling SI

The scaling group t_d can be used as a theoretical tool to assess the validity of the standard Darcy model, i.e. of Eq. (4), to describe SI. To that end, the method for calculating A must not presume the validity of Darcy's equation. To calculate A, according to Eq. (6), all the capillary-hydraulic properties need to be known. It is common practice (Gummerson et al., 1979), to obtain (k_{rw}, k_{ro}, p_c) through history-matching experimental data where the validity of Darcy's equation is assumed. Obviously, if all the (k_{rw}, k_{ro}, p_c)-sets in the data were obtained this way, then t_d would give an excellent correlation, since it is based on an exact solution of Darcy's equation. The question whether this model is applicable for counter-current SI would thus be bypassed. For the experiments performed on Berea sandstone and the synthetic porous medium, we therefore use pore scale predictions of the relative permeabilities and the capillary pressure (Valvatne and Blunt, 2004) (Fig. 1). For the synthetic material, only k_{rw} and k_{ro} have been measured. However, the curves closely resemble that for the sandstone, which indicates that the two materials have a similar pore structure, and therefore the predictions made for the sandstone sample were used. For the water–air experiments on sandstone, the measured and pore-scale predicted k_{rw} and k_{ro} were similar to that of the water–oil system (Valvatne and Blunt, 2004), and we used the same (k_{rw}, k_{ro}, J)-set as for the water–oil system, where J was scaled with the respective surface tensions σ (see Table 7). For comparison, the capillary-hydraulic properties obtained from a Darcian and a non-Darcian history match were used for some of the sandstone experiments and the diatomite experiments, respectively, and the scaled data falls onto the same curve as the data for A calculated with the pore-scale data (Fig. 4).

Since t_d contains all the information present in the Darcy model, and its validity has not been assumed to calculate A, this strongly indicates that the Darcy model is suitable for characterizing and modeling SI at the core scale, and that no additional parameters need to be introduced to model SI, contrary to recent hypotheses in engineering and physics (Barenblatt et al., 2003; Hall, 2007).

It is important to point out, however, that for viscous or gravity dominated flow the rate of change in saturation would be higher, and therefore might give rise to situations where the additional dependency of p_c on the rate of change in saturation might not be negligible any more. Also, it has been suggested that these rate effects might depend on the sample size (Bottero et al., 2011). While L_c for the experimental data used in Fig. 4 varies almost an order of magnitude, L_c is still typical for the core scale only. Hence, the question on whether rate effects should be taken into account for scales different from the core scale remains open.

Table 7
Parameter set used for the experiments in Babadagli and Hatiboglu (2007) where a water-wet Berea sandstone was used. For the experiments 11, 14 and 16, the measured S_{or} was 0.38, 0.69 and 0.57, respectively. For the experiments 12 and 18, the measured S_{or} was 0.35 and 0.65, respectively. The smallest inverse Bond number is $N_B^{-1} = 38$ for data set F-18, and thus the assumption of negligible gravity is valid.

Core	Boundary cond.	L_c (cm)	k (mD)	ϕ (–)	μ_w (Pa s)	μ_o (Pa s)	Wett.	Non-wett.	σ (mN/m)	S_0 (–)	A (m/$\sqrt{s}$)
F-11	OEO	10.16	500.0	0.21	0.001	1.8×10^{-5}	Water	Air	72.9	0	2.75×10^{-5}
F-12	OEO	15.24	500.0	0.21	0.001	1.8×10^{-5}	Water	Air	72.9	0	2.81×10^{-5}
F-14	OEO	10.16	500.0	0.21	0.001	1.8×10^{-5}	Water	Air	72.9	0	1.95×10^{-5}
F-16	OEO	5.08	500.0	0.21	0.001	1.8×10^{-5}	Water	Air	72.9	0	2.29×10^{-5}
F-16	OEO	10.16	500.0	0.21	0.001	1.8×10^{-5}	Water	Air	72.9	0	1.60×10^{-5}
F-18	OEO	15.24	500.0	0.21	0.001	1.8×10^{-5}	Water	Air	72.9	0	2.07×10^{-5}

4.5. The validity of specialized groups can be predicted with t_d

The Lucas–Washburn correlation together with some of the previously defined scaling groups are listed in Table 2. It is now apparent that previous authors (unknowingly) derived successively better approximations to the integral in Eq. (20), making the proportionality constant c increasingly simple. Many of the previously derived t_d can be obtained from simple back-of-the-envelope calculations (Appendix A) as special cases of Eq. (20).

Eq. (20) can therefore be viewed as the 'master equation' for scaling groups. It can also be used to derive new scaling groups that are tailored for a specific SI system by using an approximation for A that is appropriate for that specific case. The ability of such a special t_d to correlate a set of experiments depends on how similar c for the individual data sets are, and thus allows for a rigorous prediction of their validity and a judgment as to which parameters are negligible.

For example, c for the group by Ma et al. (1997) (Table 2, Fig. 4(a)) is similar for some of the SI experiments on sandstone ($*$) and the synthetic material ($+$) from which Ma et al. derived the correlation phenomenologically; here, the viscosity ratio is approximately one, the initial fluid content is similar and the capillary hydraulic properties were the same. These conditions result in similar functional form of F, similar integral boundaries, and the same integrand for c, respectively. Depending on which assumption is violated, five sub-groups different from the ($*$,$+$)-curve emerge: The sub-group for (i) different S_0 ($\square$), different capillary-hydraulic properties (diatomite with (ii) high ($\circ$) and (iii) low ($\circ$) μ_{nw}/μ_w), (iv) sandstone with strongly varying μ_{nw}/μ_w ($\triangleleft$), and (v) sandstone containing gas, i.e. a non-wetting phase with neglectable μ_{nw} ($*$).

5. Scaling group for mixed-wet systems

5.1. Introduction: induction times and two different S_{or}

The data set used in Figs. 4 and 5 was for widely varying conditions, but excluded one scenario: Mixed-Wettability (MW). The majority of reservoirs are not water-wet, but mixed-wet (Anderson, 1987b). The term 'mixed-wet' was coined by Salathiel (1973) for rock that contains both water-wet (WW) and oil-wet (OW) pores. On the macro-scale, MW systems are characterized by a p_c-curve that becomes negative for some $S_w > S_w^*$ (Fig. 7). If the flow is only driven by capillary forces, then SI stops although $S_w < S_{or}$, i.e. when the oil is still mobile (Anderson, 1987a,b). If $S_w > S_w^*$, then $p_c < 0$ would result into oil being imbibed until S_w is such that $p_c=0$. The difficulty of deriving a scaling group for MW-systems is therefore twofold: First, the behavior of the p_c curve (Figs. 6(b) and 7) results into two different residual oil saturations: One at which SI stops $S_{or,SI}$, and one to which S_o could
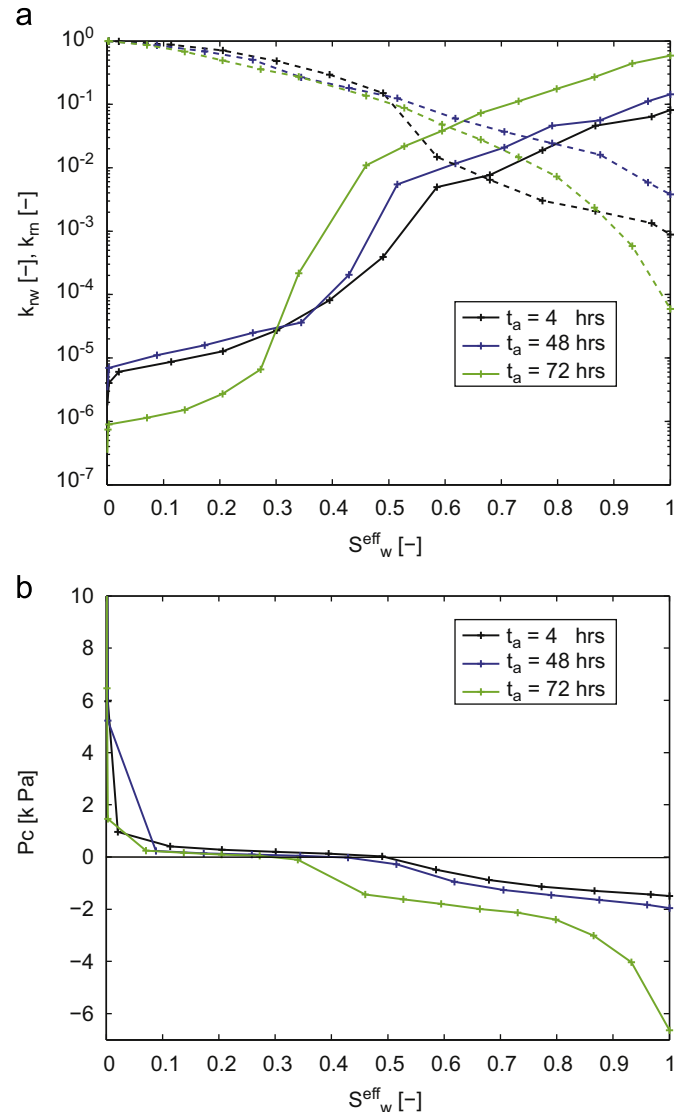
Fig. 6. Capillary-hydraulic properties vs. effective saturation for different degrees of ageing time t_a and thus different degrees of mixed wettability as predicted by pore-scale simulations (Behbahani and Blunt, 2005) for the experiments reported in Zhou et al. (2000). (a) Relative permeabilities of the aqueous (–) and non-aqueous phase (– –), and (b) p_c for different aging times t_a. Initial water saturation was $S_{wi} = 0.15$.

be reduced after a water flood, $S_{or,WF}$. So far, it is unclear, how a scaling group should account for that. Second, SI experiments for MW systems show that the time until imbibition in a MW system starts can be up to four orders of magnitude different compared to
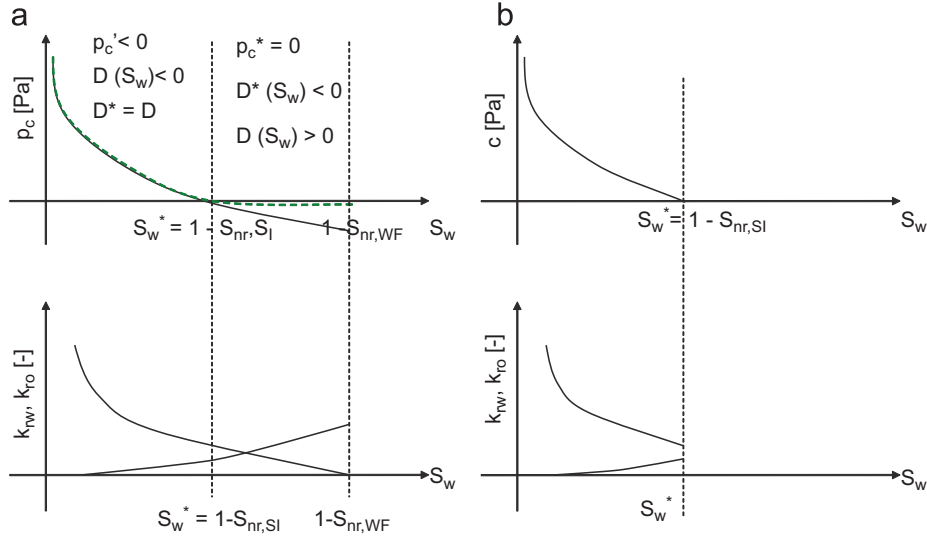
Fig. 7. Illustration of the two different methods for modifying $D(S_w)$ for MW systems. (a) 'Equivalent p_c' concept: p_c is set to zero for all $S_w > S_w^*$, but the relative permeabilities are left unchanged. (b) 'Equivalent S_{or}' concept: S_{or} is set to $S_{or} = S_{or,SI}$, and the relative permeabilities are truncated at S_w^*. For MW systems, $p_c < 0$ for some $S_w < 1 - S_{or,WF}$, such that SI stops at $S_o = S_{ro,SI}$ although the non-aqueous phase is still mobile, and would be reduced to $S_o = S_{or,WF}$ during a water-flood.

a strongly WW system (Tong et al., 2002; Xie and Morrow, 2001; Zhou et al., 2002). Previous authors tried to include this retarded behavior, known as 'induction time', by including an expression on the dynamic oil–water contact angle θ (Cil et al., 1998; Gupta and Civan, 1994; Lavi et al., 2008; Shikhmurzaev and Sprittles, 2012; Xie and Morrow, 2001; Zhou et al., 2002), as suggested in the original Lucas–Washburn equation (Lucas, 1918; Washburn, 1921).

However, Behbahani and Blunt (2005) showed that this approach is flawed. They performed pore-scale simulations on MW Berea Sandstone from which the obtained relative permeability and p_c curves. The curves were then used in a continuum-scale simulation that could reproduce the MW SI data reported in Zhou et al. (2002). The pore-scale predictions for k_{rw}, k_{ro} and p_c (Fig. 6(a), (b)) revealed that although p_c declines for increasingly MW systems, this decrease is less than one order of magnitude, and thus negligible compared to the several orders of magnitude decline in induction time. On contrast, they found that the decrease in relative permeabilities was significant (Fig. 6(a)), and concluded that a simple change in contact angle and thus p_c could not explain the induction times, and that any attempt to include the characteristics of MW systems via an effective θ would be flawed. Similarly, other pore-scale studies (Jackson et al., 2003; Valvatne and Blunt, 2004; Van Dijke and Sorbie, 2002, 2003) reveal that in the pore-space there is a wide range of advancing contact angles, and not just a single one.

In summary, if one tries to capture the behavior of a MW system via a single, effective contact angle, this angle is merely an empirical fit without any physical meaning (Anderson, 1987a; Behbahani and Blunt, 2005; Marmur, 2003). Any t_d for MW systems *must* combine the information on phase mobilities and p_c and account for the role of $S_{or,SI}$ and $S_{or,WF}$.

In the following subsection, we propose two simple methods for incorporating the information on $S_{or,SI}$ and $S_{or,WF}$ such that the complete information on the capillary-hydraulic properties is included. We validate both methods, by correlating some of the MW experiments reported in Zhou et al. (2002), compare the correlation with the one obtained for the WW experiments, and give recommendations on which method to use. Furthermore, we show how the specific group $t_{d,Beh}$ proposed by Behbahani and Blunt (2005) is another special case (Table 2) of our universal scaling group that can be used if the complete information on

k_{rw}, k_{ro} and p_c is not known. We derive the respective proportionality constant and discuss its implications for the validity of $t_{d,Beh}$.

5.2. Defining an appropriate $D(S_w)$ for MW systems

The analytical solution in Eq. (12) was derived assuming boundary conditions of some $S_{BC} > S_0$. The difference in saturation level will be 'smoothed out' as long as $D(S_w) > 0$, resulting in S_w being transported into the porous medium. In a WW system, $p_c > 0$, $dp_c/dS_w < 0$, and $D(S_w) > 0$ for $S_{wr} \leq S_w \leq 1 - S_{or}$, and $S_{or} = S_{or,SI} = S_{or,WF}$. In an experiment for a MW system, the same $S_{BC} > S_i$ would only be transported into the porous medium, as long as $p_c > 0$, i.e. as long as $S_w \leq 1 - S_{or,SI}$. However, the mathematical setting given in Eq. (12) together with (5) does not use a pressure boundary condition, and therefore for the MW system would wrongly predict that SI occurs even for $S_w > 1 - S_{or,SI}$, since for that case too $dp_c/dS_w < 0$, and thus $D(S_w) > 0$ over the whole saturation range (Fig. 6(b)). Nevertheless, an analytical solution for (7) that somehow employs pressure boundary conditions is unknown, and the setting of Eq. (7) together with (5) so far allows for the only analytical solution for a general form of $D(S_w)$ (Table 1). We therefore propose a physically motivated way of incorporating the information on p_c and the resulting end of SI at $S_w = S_{or,SI}$ into $D(S_w)$ that leaves the boundary conditions intact, such that the analytical solution given in Eq. (12) remains valid.

We propose two simple methods. In the first method (termed 'Equivalent p_c'), we define an equivalent capillary pressure curve p_c such that it agrees with the original one up to $S_w = 1 - S_{or,SI}$, but then is constantly zero (Fig. 7(a))

$$p_c^{equ.}(S_w) = \begin{cases} p_c(S_w) & \text{if } S_w \leq 1 - S_{or,SI}, \\ 0 & \text{otherwise.} \end{cases} \tag{21}$$

The relative permeabilities are left unchanged, and we also keep the residual saturation at the level, where the oil becomes immobile, i.e. $S_{or} = S_{or,WF}$. The resulting non-linear dispersion is identical with the original $D(S_w)$ up to S_w^*, and then becomes zero. Consequently, Eq. (12) correctly predicts that SI stops for $p_c < 0$, $S_w > 1 - S_{or,SI}$. WW systems are contained as a special case, since then $p_c^{equ.} = p_c$ over the whole saturation range.

In the second method (termed 'Equivalent S_{or}'), we take $S_{or} = S_{or,SI}$. That way, only the information on p_c for $S_w \leq 1 - S_{or,SI}$

Table 8
Parameter set used for experiments for different degrees of MW (Zhou et al., 2000). The rock was a Berea sandstone and the samples were aged for different times t_a yielding different MW states (Fig. 6(a), (b)). $A^{\text{Method A}}$ and $A^{\text{Method B}}$ are calculated by the two different methods for adjusting $D(S_w)$ (Section 5.2, Fig. 7). Brine was used as the aqueous phase, and Soltrol 220 was used as non-aqueous phase. The smallest inverse Bond number is $N_B^{-1} = 85$ for data set 7, and thus the assumption of negligible gravity is valid.

Core	t_a (h)	L_c (cm)	k (mD)	ϕ (−)	μ_w (Pa s)	μ_o (Pa s)	σ (mN/m)	S_0 (−)	$A^{\text{Equ. } p_c}$ (m/$\sqrt{s}$)	$A^{\text{Equ. } S_{or}}$ (m/$\sqrt{s}$)
3	4	7.62	355.0	0.215	9.67×10^{-4}	0.0398	24.2	0.151	1.25×10^{-6}	5.76×10^{-7}
7	48	7.6	365.0	0.214	9.67×10^{-4}	0.0398	24.2	0.159	5.64×10^{-7}	6.48×10^{-7}
8	72	7.6	400.0	0.2125	9.67×10^{-4}	0.0398	24.2	0.169	3.73×10^{-7}	1.25×10^{-7}

is used, and the relative permeabilities and p_c are only evaluated up to $S_w = 1 - S_{or,SI}$, but otherwise are left unchanged (Fig. 7(b)). Obviously, with this $(S_{or}, D(S_w))$-set Eq. (12) correctly predicts that SI stops at $S_w = 1 - S_{or,SI} = 1 - S_{or}$. As for the first method, WW systems are contained as a special case.

To show the validity of both approaches, we correlated the MW data reported in Zhou et al. (2002) for the cases where a suitable (k_{rw}, k_{rn}, p_c)-set is known (Table 8). We used the (k_{rw}, k_{rn}, p_c)-set derived from a pore-scale model (Behbahani and Blunt, 2005) that gave a reasonable prediction of the measured recoveries of the SI experiments.

Fig. 8 shows the result for both proposed methods, and the scaling proposed in Ma et al. (1997) as a comparison. For both methods, the data for aging time $t_a = 4$ h falls onto the same curve as the scaled data for WW conditions, showing that both methods capture the physics of the MW system. The data for $t_a = 48$ h and $t_a = 72$ h give slightly more scatter but still fall onto the same curve, i.e. t_d is able to catch the increase in induction time. The increased scatter for $t_a = 48$ h and $t_a = 72$ h stems from the pore-scale predictions of the (k_{rw}, k_{rn}, p_c)-set. For $t_a = 48$ h and $t_a = 72$ h, the agreement of the continuum scale simulations in Behbahani and Blunt (2005) degraded compared to the very good agreement achieved for $t_a = 4$ h with the experimental data, which leads to the increased scatter.

The constant A is independent of the sample geometry, and therefore is a measure for the behavior of induction times for different MW states. The SI process gets more and more retarded with increasing t_a, and A decreases in a comparable extend (Table 8). For the 'Equivalent S_{or}' method, A between $t_a = 48$ h and $t_a = 72$ h does not decrease, i.e. A for $t_a = 72$ h is slightly too big. As can be seen from Fig. 7(b), the scaling for $t_a = 48$ h gives more scatter at early times, and indicates as well, that A is slightly too big stemming again from the weaknesses in the pore-scale predictions for the hydraulic properties.

5.3. A special group for the MW case, and the influence of gravity

The 'Equivalent p_c' method was based on artificially forcing $D(S_w) = 0$ for $S_w > 1 - S_{or}$ while the 'Equivalent S_{or}' method was obtained by simply truncating information at a point where an SI process would naturally stop. Therefore, while both give comparable results, we recommend the 'equivalent S_{or}' method since it is based on all – but exclusively on – the information present in an SI system, and therefore has a sound physical justification.

For an MW system, the oil mobility $\lambda_o = k_{ro}/\mu_o$ exceeds the water mobility $\lambda_w = k_{rw}/\mu_w$ by orders of magnitudes (Fig. 6(a)), i.e. $\lambda_w(S_w) \ll \lambda_o(S_w)$ for $S_w < 1 - S_{or,SI}$. This can be used to derive the scaling group for MW systems given in Behbahani and Blunt (2005) (Appendix A) as another special case of Eq. (20),

$$t_{d,Beh} = \sqrt{\frac{K}{\phi} \frac{\sigma}{L_c^2}} \lambda_w^* t \tag{22}$$

which provides the (so far missing) physical justification for that group. From the proportionality constant c for $t_{d,Beh}$ (Table 2, Appendix A) the following predictions for the validity of $t_{d,Beh}$ can be made: A good correlation can be achieved independent of viscosity ratios, but for systems with the same initial fluid distribution, and the same capillary hydraulic properties. In particular: The correlation will be satisfactory only if the experiments were obtained under the same degree of mixed wettability.

In the aforementioned analysis, SI due to gravity segregation was ignored.

Schechter et al. (1994) proposed the inverse Bond number N_B^{-1} for investigating the relative contribution of gravity and capillary forces. N_B^{-1} is defined as

$$N_B^{-1} = C \frac{\sigma \sqrt{\phi/K}}{\Delta \rho g H}, \tag{23}$$

where $\Delta \rho$ is the density difference between the oil and the water phases, g is the gravity constant, H is the height, and $C = 0.4$ (Schechter et al., 1994). For large density differences, N_B^{-1} tends towards zero, and gravity dominates. Similarly, if N_B^{-1} becomes large, the flow is capillary dominated. Schechter et al. (1994) suggested that for $N_B^{-1} > 5$ the flow is capillary dominated, and for $N_B^{-1} \ll 1$ gravity dominated. For all experiments considered in this work, $N_B^{-1} > 5$ (Tables 3–8), and thus the assumption of capillary dominated flow is valid. Hence, we do not discuss gravity segregation any further, but merely remark that it usually is taken into account by defining (Xie and Morrow, 2001)

$$t_{d,complete} = t_{d,SI} + t_{d,grav} \tag{24}$$

and

$$t_{d,grav} = t \frac{K/\phi}{L_H \sqrt{\mu_w \mu_o}} \Delta \rho g, \tag{25}$$

where L_H is the characteristic height. In how far this approach can describe recovery if instabilities, like gravity driven fingers (Cueto-Felgueroso and Juanes, 2008) or gas-gravity drainage (Di Donato et al., 2007), occur or whether $t_{d,grav}$ gives a good correlation for experiments where the parameters are varied as suggested by Section 2, remains open.

6. Applications: dual-porosity models and the first general form of Aronofsky's model

Scaling groups are essential in any situation where SI needs to be understood, described and modeled which is the case for a wide range of geophysical and petrophysical applications. Examples include the upscaling of laboratory data (Morrow and Mason, 2001), the characterization of a porous rock (Jadhunandan and Morrow, 1991; Marmur, 2003; Morrow et al., 1994), feasibility studies for geothermal reservoirs (Li and Horne, 2009) and even applications not related to petrophysical applications at all, like the water uptake and following germination process of plant
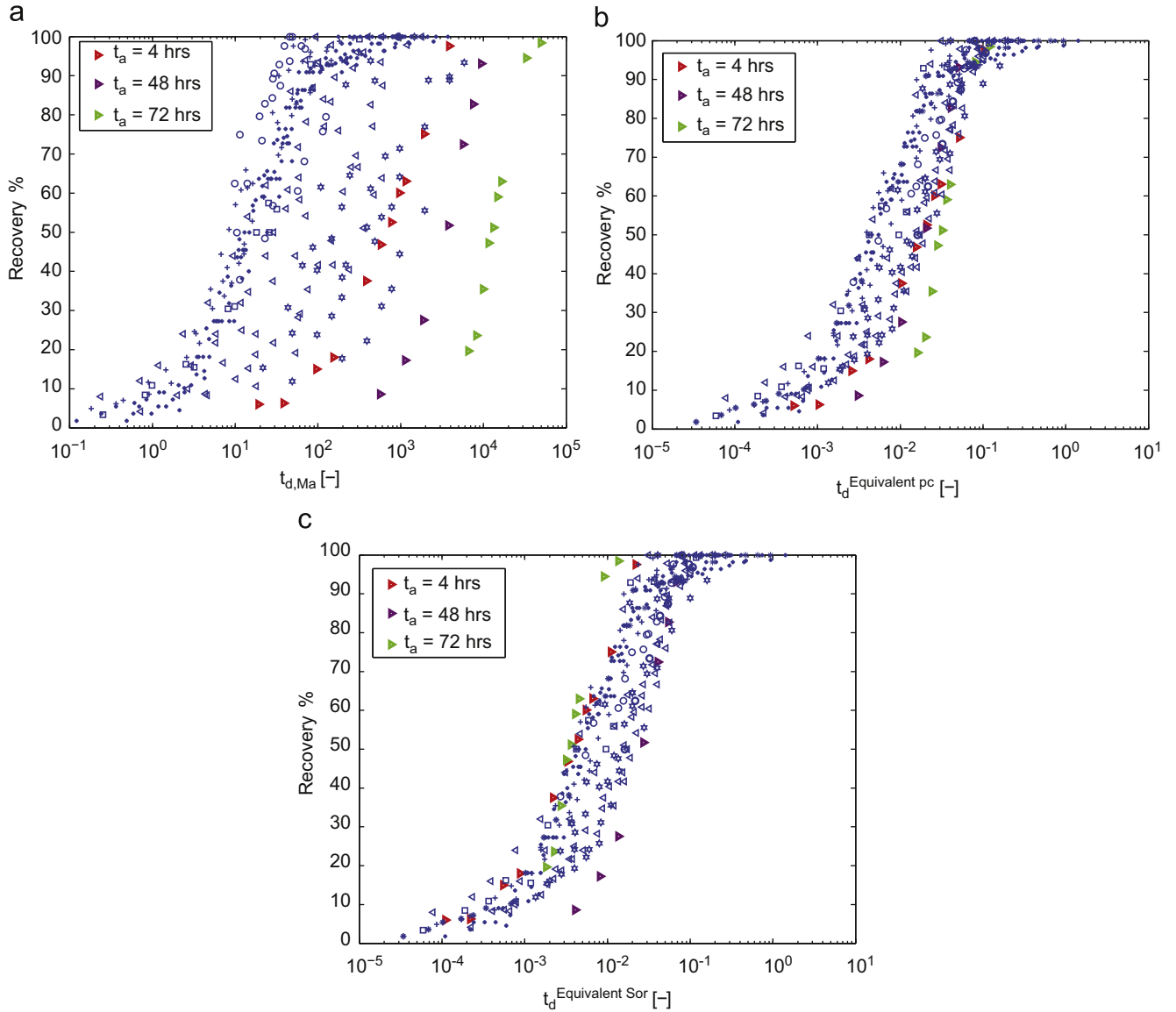
Fig. 8. Scaling of experimental data for the MW case. The data in blue is for the WW case and serves as reference. (a) Time scaled according to Ma et al. (1997). The scaled MW data lead to an even further increase in scatter. (b) Time scaled according to the 'Equivalent p_c' concept, and (c) according to the 'Equivalent S_{or}' concept. The results for both methods fall onto the same curve as for the WW data but the 'Equivalent S_{or}' only used information naturally present in an SI system. It therefore should be preferred although for the data set presented here it results in more scatter than the Equivalent p_c concept. Pore-scale predictions (Behbahani and Blunt, 2005) for k_{rw}, k_{ro} and p_c were used. The best (k_{rw},k_{ro},p_c)-set was obtained for $t_a=4$ h, which therefore gives the best correlation while $t_a=48$ h and $t_a=72$ h lead to more scatter. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

seeds (Schmid and Geiger, 2012; Finch-Savage et al., 2005; Finch-Savage and Leubner-Metzger, 2006).

A key application is dual-porosity models where scaling groups build the center piece. Dual-porosity models are field-scale representations of fractured reservoirs that separate the subsurface model into stagnant, low permeability regions (the rock matrix blocks) and high-permeability regions (the fracture network), and the fluid transfer between the two regions is modeled by some transfer function T (Warren and Root, 1963; Kazemi et al., 1976). Since many expressions of T rest upon an analytical expression for SI and/or a scaling group t_d, the numerous transfer rates for SI that have been proposed to date (Abushaikha and Gosselin, 2008; Babadagli and Zeidani, 2004; Cil et al., 1998; Kazemi et al., 1992; Unsal et al., 2010) experience similar challenges as the scaling groups themselves: It is unclear how a model can describe the heterogeneity in a reservoir that stems from differences in initial water content, wettability and

phase mobility effects, and how SI data obtained for a certain viscosity range translates into the conditions found in the field.

Since our t_d overcomes all these challenges, it is easy to derive an improved transfer function, as well. In a dual-porosity model, the mass conservation equations for the fracture and the matrix are

$$\phi_f \frac{\partial S_{wf}}{\partial t} + q_t \cdot \nabla f_f = -T,$$

$$\phi_m \frac{\partial S_{wm}}{\partial t} = T, \qquad (26)$$

where the subscripts m and f stand for matrix/stagnant region and fracture/flowing region, respectively. To obtain an expression for T based on t_d we use that the recovery R satisfies

$$\frac{R}{R_\infty} = \frac{S_{wm}-S_{wmi}}{1-S_{orm}-S_{wim}}, \qquad (27)$$

where R_∞ is the final recovery. Aronofsky et al. (1958) proposed an exponential model for R as

$$R = R_\infty(1 - e^{-\alpha t_d}). \tag{28}$$

From fitting the recovery curve in Fig. 8, we obtain $\alpha \approx 70$. From Eqs. (26) and (27), we obtain (Di Donato et al., 2007)

$$\phi_m \frac{\partial S_{wm}}{\partial t} = T = \alpha \frac{t_d}{t}(1 - S_{orm} - S_{wm})$$
$$= \alpha \tau_c(1 - S_{orm} - S_{wm}). \tag{29}$$

The significance of this result is the following: The value for α is obtained for *arbitrary* experimental conditions, i.e. α is *independent* of wettability states, rock material and so forth. The influence of capillary-hydraulic properties and characteristic length scales is captured in τ_c. Therefore, the model given in Eq. (29) is the first one, where α does not have to be 're-fitted' to different experimental conditions as has been the common strategy so far (Abushaikha and Gosselin, 2008; Babadagli and Zeidani, 2004; Cil et al., 1998; Di Donato et al., 2007; Kazemi et al., 1992; Unsal et al., 2010). It therefore simply can be incorporated into any dual-porosity model without the need for further experiments that would be necessary for obtaining a modified α. Furthermore, Eq. (29) shows that an exponential Aronofsky model for recovery is capable to incorporate the physics of SI if t_d is appropriately chosen, in contrast to what has been claimed before (Chen et al., 1995).

t_d is based on laboratory results where the core used was surrounded by a constant saturation. In fractured systems, the saturation changes depending on the speed of the fracture flow which leads to different fracture flow regimes (Rangel-German and Kovscek, 2006, 2002; Fernø et al., 2011). Therefore, accounting for heterogeneities in petrophysical properties alone might not be sufficient, and additionally the characteristic lengths might have to be modified (Rangel-German and Kovscek, 2006, 2002).

7. Conclusions

While the $\sqrt{t}$-scaling for SI has been known for more than 90 years, the properties of this scaling were unknown. Here, we derived the first scaling group t_d that rigorously incorporates the influence of all parameters present in the two-phase Darcy formulation. This paper allows for the following conclusions:

1. *The analytical solution is the capillary analogue to the Buckley–Leverett equation.* The knowledge of a general analytical solution is crucial for deriving appropriate scaling groups and other applications. Based on Schmid et al. (2011, Appendix A) we demonstrated that the analytical solution proposed in McWhorter and Sunada (1990) is general (Fig. 4) and can be viewed as the capillary analogue to the Buckley–Leverett solution; this should finalize the long search (Table 1) for a general solution. The constant A given by Eq. (8) describes a porous medium's ability to imbibe.
2. *SI is best characterized by the cumulative inflow and not by the frontal movement.* Based on the analytical solution two scaling groups for SI can be derived – one based on the cumulative inflow, one based on the frontal movement of the wetting front – that comprise all the information present in the Darcy model. No assumptions other than those needed for Darcy's model are made, and no fitting parameters need to be introduced. The group based on the cumulative wetting phase gives an excellent correlation which is superior to a characterization based on the frontal movement, contrary to what has been speculated by Handy (1960) and subsequent authors.
3. *Our group serves as a 'master equation' that predicts the validity of specialized groups.* Due to its generality, many of the previously defined groups are contained as special cases, and the validity of a special group can rigorously be predicted through the proportionality constant c. Any new specific scaling group with a sound physical foundation necessarily is a special case of our group.
4. *Darcy's model can describe SI.* Our group comprises all the information present in the Darcy model, and to calculate necessary parameters (e.g. k_{rw}), the validity of Darcy's model was not presumed. The correlated data falls onto one curve with little scatter which indicates that the Darcy model can characterize SI and no further parameters are necessary.
5. *A transfer function can be derived that is suitable for arbitrary petrophysical properties.* An exponential model for the transfer can be fitted that correlates 45 published water–oil and water–air SI studies for water-wet systems, different degrees of mixed-wettability, a wide range of viscosity ratios, initial water content, different porous media and different boundary conditions. This is the first general transfer function, and readily can be used in any dual porosity simulator.
6. *Limitations.* The group t_d has been derived making the standard assumptions used for analytical modes (Section 3.1). In particular, viscous and gravity forces were ignored. Whether the presence of these forces can be accounted for through, e.g. Eq. (24), and whether 'rate effects' in p_c would then have to be considered, is open. Also, t_d is based on laboratory results where the core used was surrounded by a constant saturation. If and how t_d should be changed for, e.g. fractured systems, where the saturation changes depending on the speed of the fracture flow, is open.

Appendix A

In this appendix, we show how the different specialized groups can be obtained from t_d if the respective assumptions used for the specialized groups are evoked (Table 2).

A.1. The group of Lucas and Washburn (Lucas, 1918; Washburn, 1921)

The Lucas–Washburn equation follows if the original assumption (Lucas, 1918; Washburn, 1921) of an inviscid non-wetting phase is invoked, i.e. $\mu_o \ll \mu_w$. Then

$$\frac{\lambda_w \lambda_{nw}}{\lambda_t} = \frac{1}{\sqrt{\mu_w \mu_{nw}}} \frac{\sqrt{k_{rw} k_{ro}}}{\sqrt{M} + 1/\sqrt{M}} \approx \frac{k_{rw}}{\mu_w},$$

since $\sqrt{M} \ll 1/\sqrt{M}$, where $M = \mu_n k_{rw}/\mu_w k_{ro}$, and the capillary radius is substituted by the Leverett-radius (Bear, 1972), $r = \sqrt{K/\phi}$. This then gives

$$F(S_w) \approx 1 - \left(\int_{S_w}^{S_i} \frac{(\beta - S_w)k_{rw}p_c'}{F(S_w)} d\beta \right) \cdot \left(\int_{S_i}^{S_0} \frac{(S_w - S_i)k_{rw}p_c'}{F(S_w)} dS_w \right)^{-1}$$
$$\tag{A.1}$$

and thus

$$\left(\frac{A}{\phi L_c}\right)^2 \approx \sigma \sqrt{\frac{K}{\phi}} \frac{1}{\mu_w L_c^2} \int_{S_i}^{S_0} \frac{(S_w - S_i)k_{rw}J'}{F(S_w)} \, dS_w \int_{S_i}^{S_0} \frac{(S_w - S_i)k_{rw}J'}{F(S_w)} \, dS_w$$

$$= \sigma \sqrt{\frac{K}{\phi}} \frac{1}{\mu_w L_c^2} \cdot c. \tag{A.2}$$

A.2. The group of *Rapoport (1955), Mattax and Kyte (1962)*

Mattax and Kyte (1962) used the scaling laws developed by Rapoport (1955) to derive a scaling group for the case where $\mu_o = \mu_w$. This yields

$$\frac{\lambda_w \lambda_o}{\lambda_t} = \frac{1}{2\mu_w} \sqrt{k_{rw}k_{ro}}$$

which gives

$$F(S_w) = 1 - \left(\int_{S_w}^{S_i} \frac{(\beta - S_w)\sqrt{k_{rw}k_{ro}}p_c'}{F(S_w)} \, d\beta \right) \cdot \left(\int_{S_i}^{S_0} \frac{(S_w - S_i)\sqrt{k_{rw}k_{ro}}p_c'}{F(S_w)} \, dS_w \right)^{-1}$$

and thus

$$\left(\frac{2A}{\phi L_c}\right)^2 = \sigma \sqrt{\frac{K}{\phi}} \frac{1}{L_c^2 \mu_w} \int_{S_i}^{S_0} \frac{(S_w - S_i)\sqrt{k_{rw}k_{ro}}J'}{F} \, dS_w$$

$$= \sigma \sqrt{\frac{K}{\phi}} \frac{1}{L_c^2 \mu_w} \cdot c. \tag{A.3}$$

A.3. The group of *Ma et al. (1997)*

The Ma et al. (1997)-group uses the assumption that $\mu_w \approx \mu_o$ which together with

$$\frac{\lambda_w \lambda_o}{\lambda_t} \approx \frac{1}{2\sqrt{\mu_w \mu_o}} \sqrt{k_{rw}k_{ro}} \tag{A.4}$$

leads to

$$F(S_w) \approx 1 - \left(\int_{S_w}^{S_i} \frac{(\beta - S_w)\sqrt{k_{rw}k_{ro}}p_c'}{F(S_w)} \, d\beta \right) \cdot \left(\int_{S_i}^{S_0} \frac{(S_w - S_i)\sqrt{k_{rw}k_{ro}}p_c'}{F(S_w)} \, dS_w \right)^{-1}.$$

Therefore,

$$\left(\frac{2A}{\phi L_c}\right)^2 \approx \sigma \sqrt{\frac{K}{\phi}} \frac{1}{L_c^2 \sqrt{\mu_w \mu_o}} \int_{S_i}^{S_0} \frac{(S_w - S_i)\sqrt{k_{rw}k_{ro}}J'}{F} \, dS_w$$

$$= \sigma \sqrt{\frac{K}{\phi}} \frac{1}{L_c^2 \mu_w} \cdot c. \tag{A.5}$$

Note that the influence of viscosities in Eq. (A.4) could only be eliminated because matched viscosities were assumed. If that assumption does not hold for all the data sets that are correlated, then F does not have the form given above, and c is different for the different data sets.

A.4. The group of *Zhou et al. (2002)*

Zhou et al. (2002) used the assumption $(\lambda_w \lambda_o / \lambda_t)(S_w) \approx (\lambda_w \lambda_o / \lambda_t)^*$ which immediately yields

$$\left(\frac{2A}{\phi L_c}\right)^2 \approx 2\sigma \sqrt{\frac{K}{\phi}} \frac{1}{L_c^2} \left(\frac{\lambda_w \lambda_o}{\lambda_t}\right)^* \int_{S_i}^{S_0} \frac{(S_w - S_i)J'}{F} \, dS_w$$

$$= 2\sigma \sqrt{\frac{K}{\phi}} \frac{1}{L_c^2} \left(\frac{\lambda_w \lambda_o}{\lambda_t}\right)^* \cdot c. \tag{A.6}$$

A.5. The group of *Behbahani and Blunt (2005)*

Behbahani and Blunt (2005) considered MW systems, where they found from pore-scale studies that $\lambda_w \ll \lambda_o$ and consequently

$$\frac{\lambda_w \lambda_o}{\lambda_t} \approx \lambda_w.$$

If furthermore is assumed that (although this assumption was not stated explicitly) $\lambda_w(S_w) \approx \lambda(S_w^*)$, then

$$\left(\frac{2A}{\phi L_c}\right)^2 \approx 2\sigma \sqrt{\frac{K}{\phi}} \frac{1}{L_c^2} \int_{S_i}^{S_0} \frac{(S_w - S_i)\lambda_w(S_w)J'}{F} \, dS_w$$

$$\approx 2\sigma \sqrt{\frac{K}{\phi}} \frac{1}{L_c^2} \lambda_w^* \int_{S_i}^{S_0} \frac{(S_w - S_i)J'}{F} \, dS_w$$

$$= 2\sigma \sqrt{\frac{K}{\phi}} \frac{1}{L_c^2} \lambda_w^* \cdot c. \tag{A.7}$$

A.6. The group of *Li and Horne (2006)*

Li and Horne (2006) considered piston-like flow, i.e. they assumed that $F(S_w) = q_w(x^*,t)/q_w(0,t)$ and $p_c(S_w)' = p_c/S_w$. $F(x,t)$ can be re-written in terms of saturation as $F(S_w) = q_w(S_w^*)/q_w(S_i) = const. = 1/c$. Consequently

$$\left(\frac{2A}{\phi L_c}\right)^2 \approx 2\sigma \sqrt{\frac{K}{\phi}} \frac{1}{L_c^2} \int_{S_i}^{S_0} \frac{(S_w - S_i)\frac{\lambda_w \lambda_o}{\lambda_t}J'}{F} \, dS_w$$

$$= 2\sigma \sqrt{\frac{K}{\phi}} \frac{1}{L_c^2} c \int_{S_i}^{S_0} (S_w - S_i)\frac{J}{(S_w - S_i)} \, dS_w$$

$$= 2\sigma \sqrt{\frac{K}{\phi}} \frac{1}{L_c^2} c \int_{S_i}^{S_0} J \, dS_w$$

$$\approx 2\sigma \sqrt{\frac{K}{\phi}} \frac{1}{L_c^2} J^*(S_0 - S_i) \cdot c. \tag{A.8}$$

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